University of Zurich
Institute for Empirical Research in Economics

Advanced Portfolio Theory

NHH-Bergen

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IEW
Universität Zürich

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10. Evolutionary Portfolio Theory: Survival of the Fittest at Wall Street

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- Timing: Div yields
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Characteristics of Financial Markets

- Huge Heterogeneity of Behaviour!
- Endogeneity of Returns
- Success and Failures of Investors
- Rapid Innovation Process of Strategies

There is no portfolio theory or asset pricing model that is able to cope with these important features.

Veblen (1898)

Veblen, T. (1898) " Why Is Economics not an Evolutionary Science?"

The Quarterly Journal of Economics 12: 373-397.

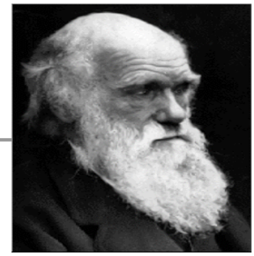
Marshall (1948)

The Mecca of economics lies in economic biology rather than economic mechanics.

Alfred Marshall, Principles of Economics, 8th edition,

MacMillan, 1948.

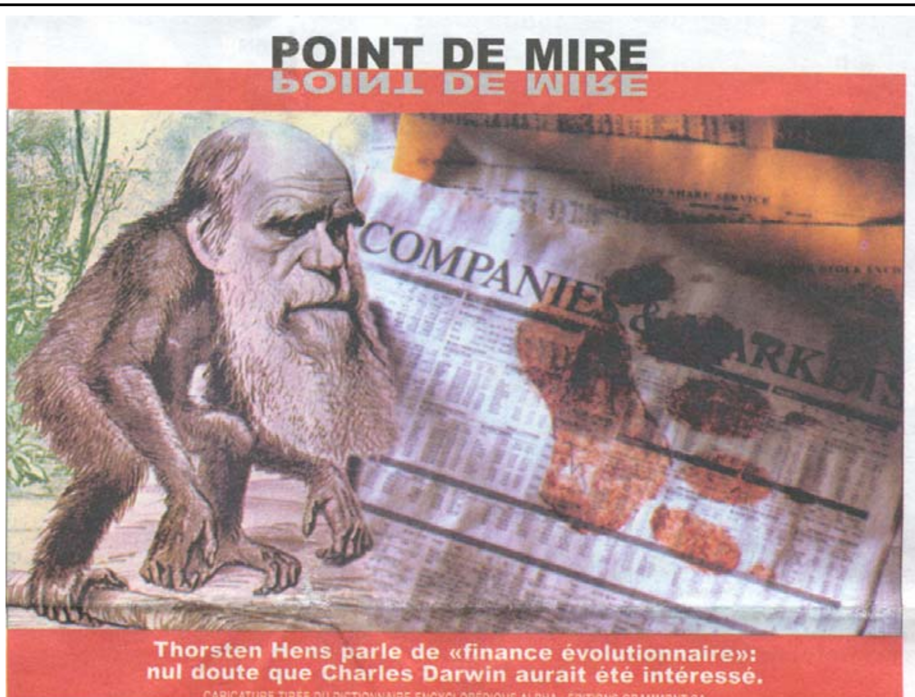
Evolutionary Portfolio Theory



„Survival of the Fittest at Wall Street."

Evolutionary Biology as a New Paradigm for Finance

Biology:	**Finance:**
Strategy	Portfolio rule
Resources	Market capital
Selection	Gains / losses
Mutation	Innovation



Evolutionary Portfolio Theory

Model:

Lucas (1978) Tree Model: $\omega_t \in \Omega, \quad \omega^t = (\omega_0, ..., \omega_t)$

$\lambda_t^{i,k}$ $i =$ 1,...,I strategies P prob measure

$k =$ 1,...,K assets

$k = 0$ consumption good

Evolution of Wealth:

$$w_{t+1}^i(\omega^{t+1}) = \left\{ \sum_{k=1}^{K} \left[\frac{D_{t+1}^k(\omega_{t+1}) + P_{t+1}^k(\omega^{t+1})}{P_t^k(\omega^t)} \right] \lambda_t^{i,k}(\omega^t) \right\} w_t^i(\omega^t)$$

Market Interaction

Equilibrium Prices:

$$P_t^k(\omega^t) = \sum_{i=1}^{I} \lambda_t^{i,k}(\omega^t) w_t^i(\omega^t)$$

„The price of asset k is the wealth-average of the strategies` portfolio share for asset k.“

Analysis of the evolution of **relative** wealth

Evolution of **relative** wealth:

$$r_{t+1}^{i}(\omega^{t+1}) = \left\{ \sum_{k=1}^{K} \left[\frac{d_{t+1}^{k}(\omega_{t+1}) + q_{t+1}^{k}(\omega^{t+1})}{q_{t}^{k}(\omega^{t})} \right] \lambda_{t}^{i,k}(\omega^{t}) \right\} r_{t}^{i}(\omega^{t})$$

where

$$q_{t}^{k}(\omega^{t}) = \frac{P_{t}^{k}(\omega^{t})}{\sum_{i} w_{t}^{i}(\omega^{t})} \qquad d_{t+1}^{k}(\omega_{t+1}) = \frac{D_{t+1}^{k}(\omega_{t+1})}{\sum_{i} w_{t}^{i}(\omega^{t})}$$

$$r_{t}^{i}(\omega^{t}) = \frac{w_{t}^{i}(\omega^{t})}{\sum_{i} w_{t}^{i}(\omega^{t})}$$

Market Interaction

Equilibrium Prices:

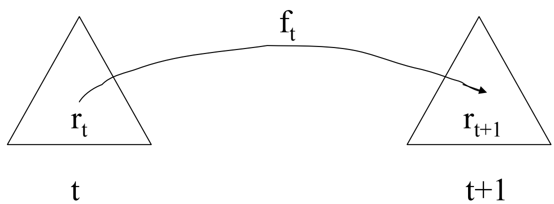
$$q_{t}^{k}(\omega^{t}) = \sum_{i=1}^{I} \lambda_{t}^{i,k}(\omega^{t}) r_{t}^{i}(\omega^{t})$$

„The price of asset k is the **relative** wealth-average of the strategies` portfolio share for asset k.“

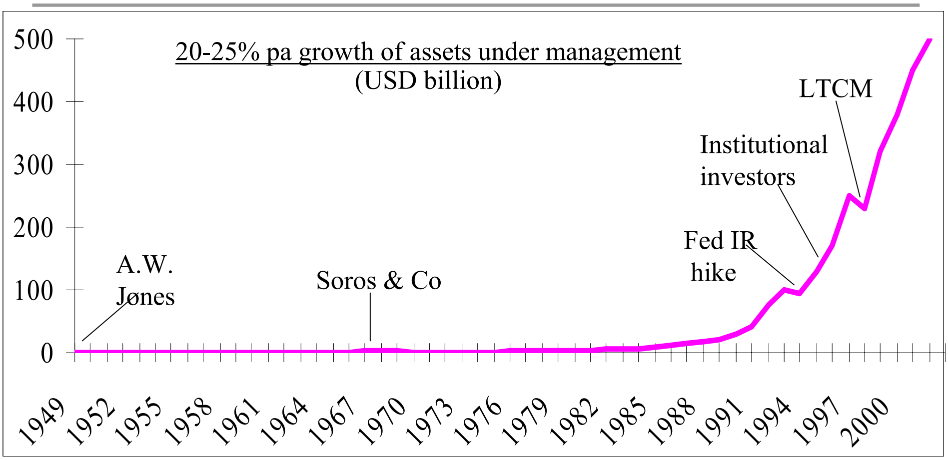
Evolution of Trading Strategies

Random Dynamical System:

$$r_{t+1}(\omega^{t+1}) = f_t\ (\omega^{t+1}, r_t)$$

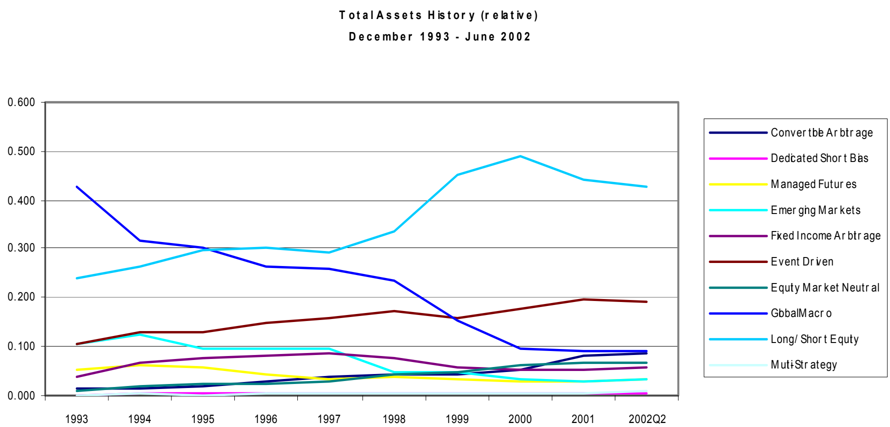


Example: Hedge Funds



Source: Harcourt estimates, MAR, TASS, various industry sources

Example: Hedge Funds



Simulation Analysis

Example: Asset Allocation on DJIA

1) Known: Exogenous Dividend Process

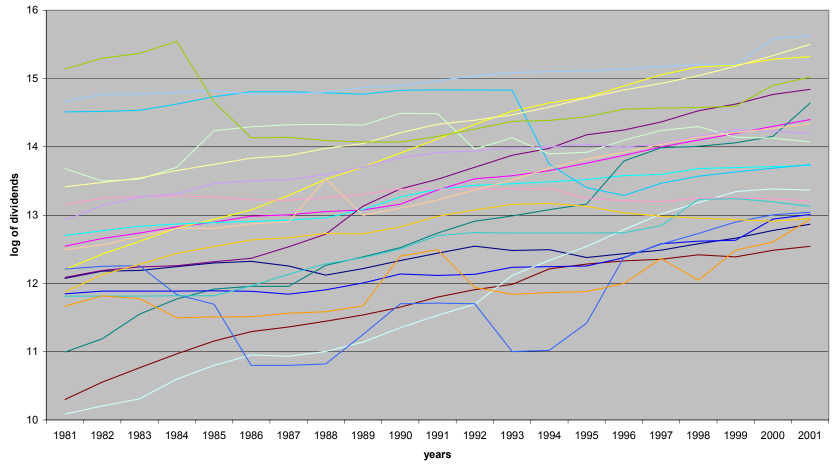
(based on Data 1981-2000, assumed to be stationary)

2) Unknown: Endogenous Return Process

(price process is determined by market equilibrium)

3) Market Selection of Simple Portfolio Rules

The DJIA-Dividend Process



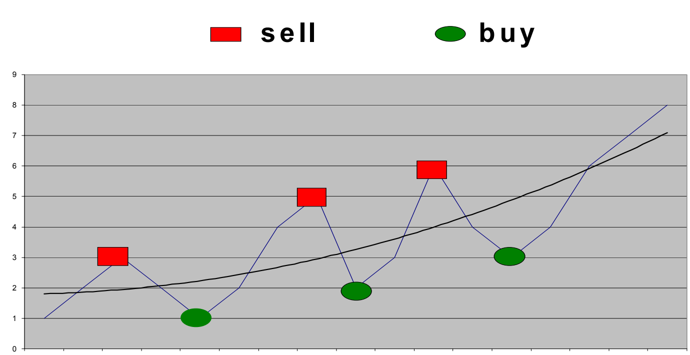
$$D_t^k(\omega^t)$$

Asset Allocation on DJIA

Simple Strategies:

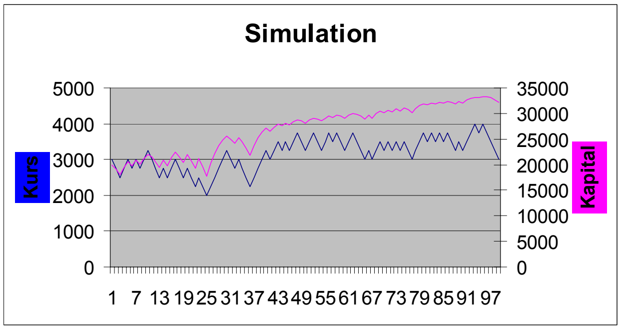
Hold portfolio weights $\lambda^{i,k}$ fix over time.

Rebalance units of assets when prices change:



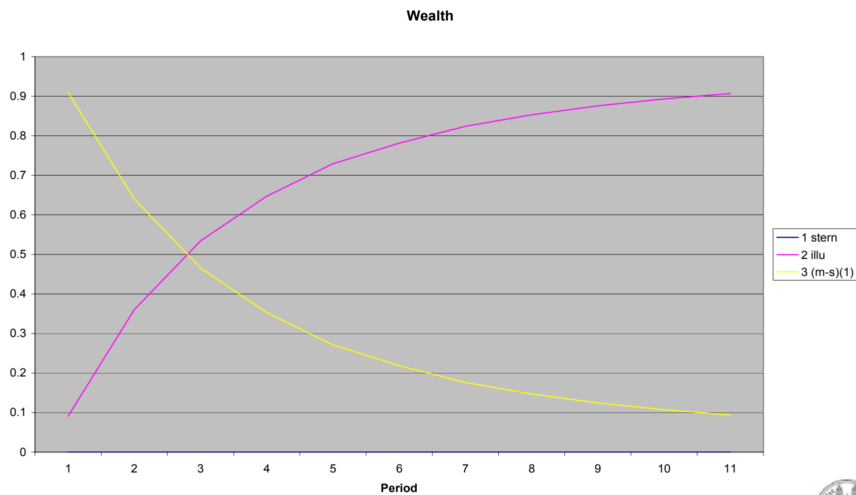
Volatility Pumping

- Th strategy

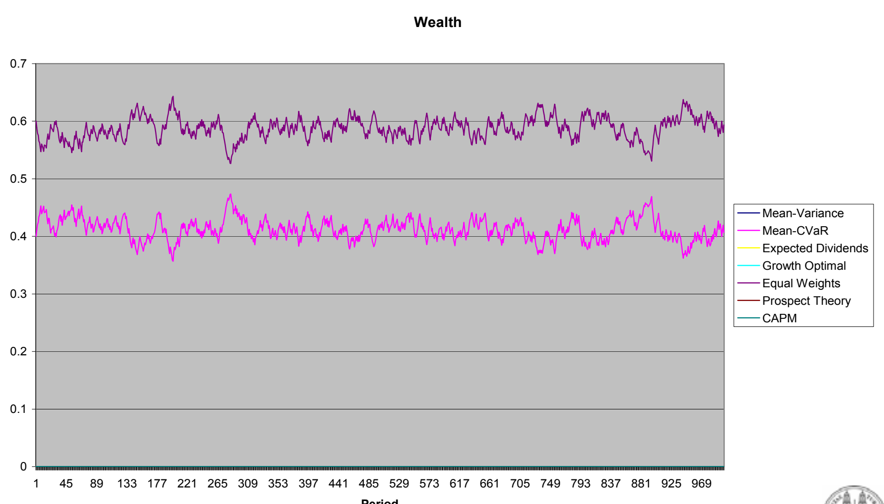


Show ML-Program

Mean-Variance is driven out by (1/n)



CVaR and (1/n) stochastically hold a balance



Mathematical Analysis

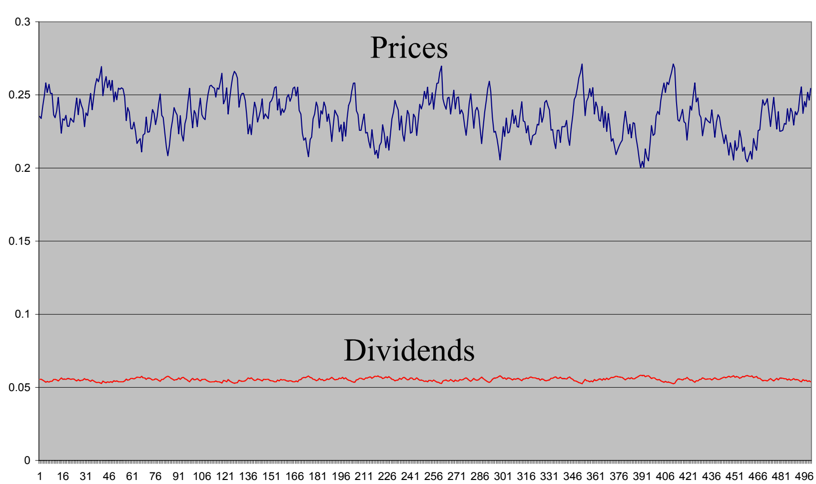
Key Term:

$$g(\lambda^{i},\lambda^{n}) = E_p \ln\left(1-\lambda_0+\lambda_0\sum_{k=1}^{K} d^{k}_{(\omega)}\,\frac{\lambda^{i,k}}{\lambda^{n,k}}\right)$$

the exponential growth rate of strategy i in a monomorphic population n.

λ_0 consumption rate (identical across rules)

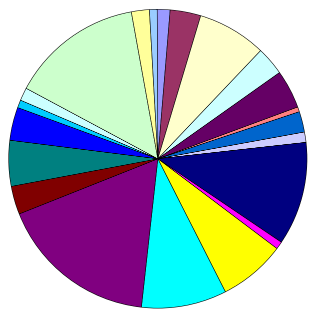
Excess Volatility in Asset Prices



The Evolutionary Portfolio Rule

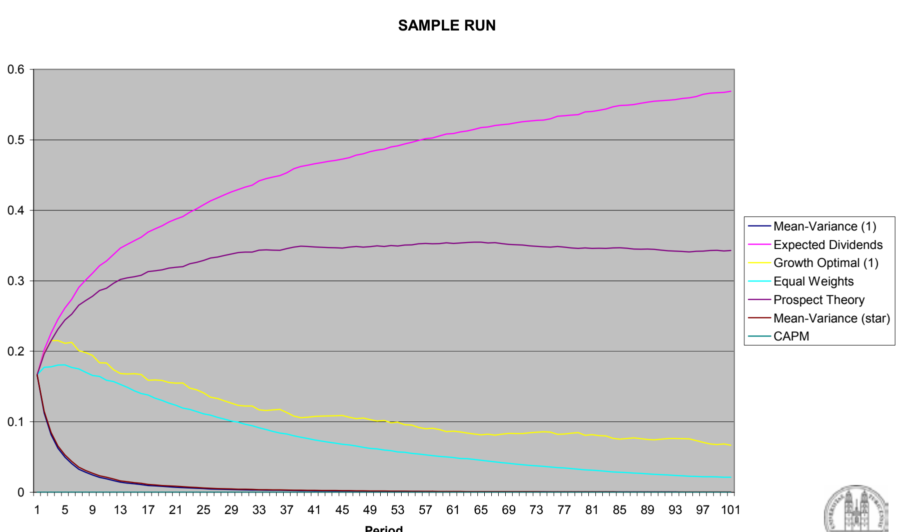
$$\overset{*}{\lambda}{}^{k} = (1-\lambda_0)E_p d^{k}_{(\omega)}$$

Expected Dividends Portfolio $\overset{*}{\lambda}$

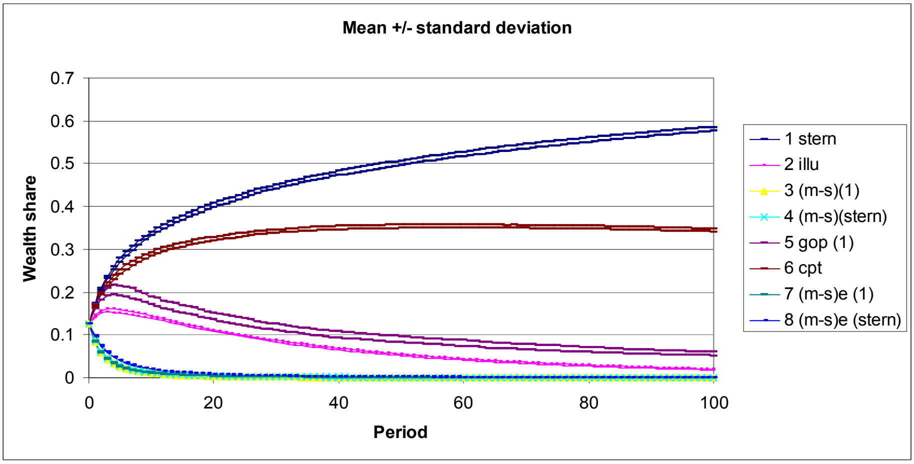


Has highest growth rate against itself

$\overset{*}{\lambda}$ is single survivor



Confidence Intervals



Sciubba (1998): Staying Alive

"This claim has been opposed by Merton and Samuelson and Goldman.

In particular, Merton and Samuelson's critique stressed the obvious contradiction which lies in arguing that rational traders should maximise a utility function which is different from their own. The evolutionary framework allows us to contribute to the debate suggesting that maximising a logarithmic utility function might not make you happy, but will definitely keep you alive!"

(Sciubba, *The Evolution of Portfolio Rules and the Capital Asset Pricing Model, 1998*)

Diversification Theorem

Evstigneev, Hens, Schenk-Hoppé (2002)

Suppose dividends **d** follow a stationary ergodic process **and** consider λ **simple** (constant in time)

Then $\overset{*}{\lambda}{}^{k} = (1-\lambda_0)E_p d^k_{(\omega)}$ is the single survivor of the market selection process.

Mean-Variance vs Log

- Adapted strategies
- Martin Stalder

Mathematical Analysis

Key Term:

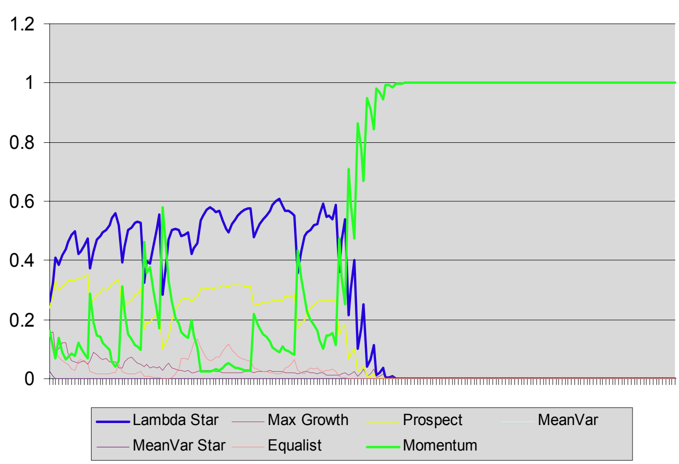
$$g(\lambda^{i},\lambda^{n}) = E_p \ln\left(1-\lambda_0+\lambda_0\sum_{k=1}^{K} d^{k}_{(\omega)}\frac{\lambda^{i,k}}{\lambda^{n,k}}\right)$$

the exponential growth rate of strategy i in a monomorphic population n.

λ_0 consumption rate (identical across rules)

Results for Adapted Strategies

Evolution of relative Strategy Wealth



Evolutionary Stable Investment

Theorem: Evstigneev, Hens, Schenk-Hoppé (2002)

Suppose dividends **d** follow a stationary ergodic process

and consider λ adapted.

Then $\overset{*}{\lambda}{}^k = (1-\lambda_0) E_p d^k_{(\omega)}$

is the unique

evolutionary stable strategy.

g(.,.)	$\overset{*}{\lambda}$	$\hat{\lambda}$	$\overset{\approx}{\lambda}$
$\overset{*}{\lambda}$	0	<0	>0
$\hat{\lambda}$	<0	0	>0
$\overset{\approx}{\lambda}$	<0	>0	0

Corollaries

1) Every underdiversified strategy *n* can easily be invaded:

$$\lambda^n_k = 0 \Rightarrow g(\lambda^i, \lambda^n) > 0 \quad \text{for some}$$

2) CAPM keeps its relative position (imitation strategy):

$$\lambda^{CAPM}_{k,t} = q^k_t \qquad q^k_t = \sum_{i=1}^{I} \lambda^{i,k}_t r^i_t$$

Mathematical Analysis

Key Term:

$$g(\lambda^{i},\lambda^{n}) = E_p \ln\left(1-\lambda_0+\lambda_0\sum_{k=1}^{K} d^{k}_{(\omega)}\frac{\lambda^{i,k}}{\lambda^{n,k}}\right)$$

the exponential growth rate of strategy i in a monomorphic population n.

λ_0 consumption rate (identical across rules)

Timing

- Lambda0 according to Div-yield-interest rate
- Show Backtracking Results

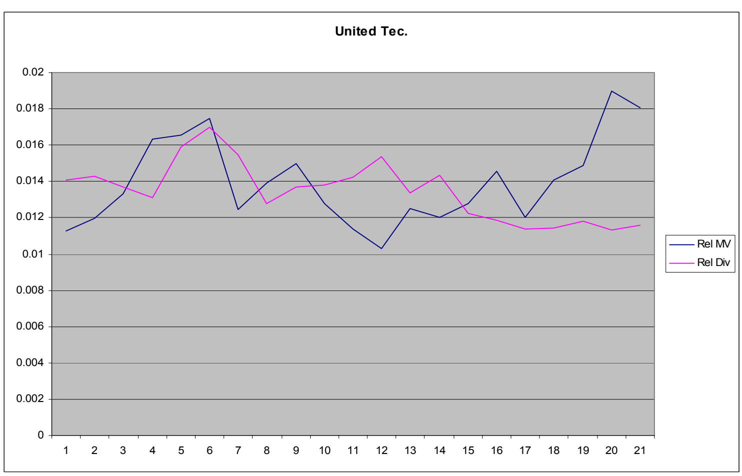
Back-testing the Evolutionary Portfolio Rule

Performance of λ^* ***without any market interaction***.

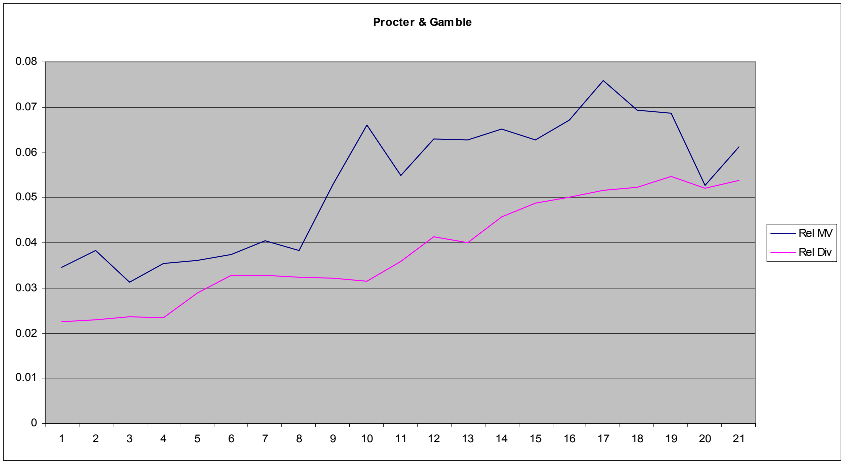
λ^* is a value investment rule.

Value investment works due to excess volatility.

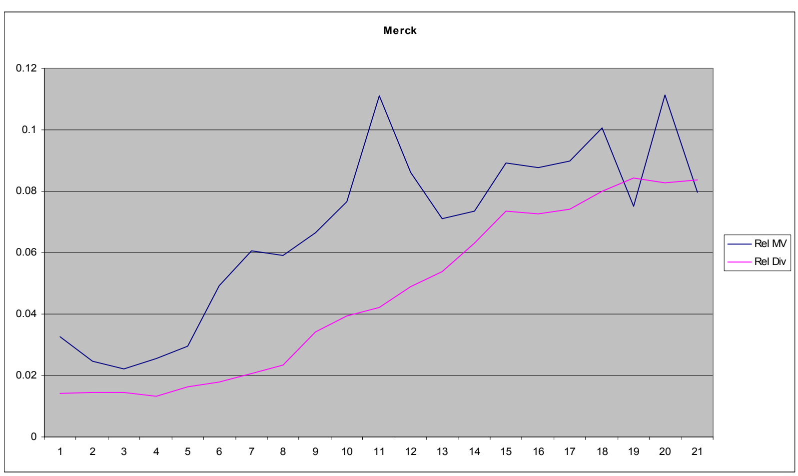
Relative MV follow relative DIV



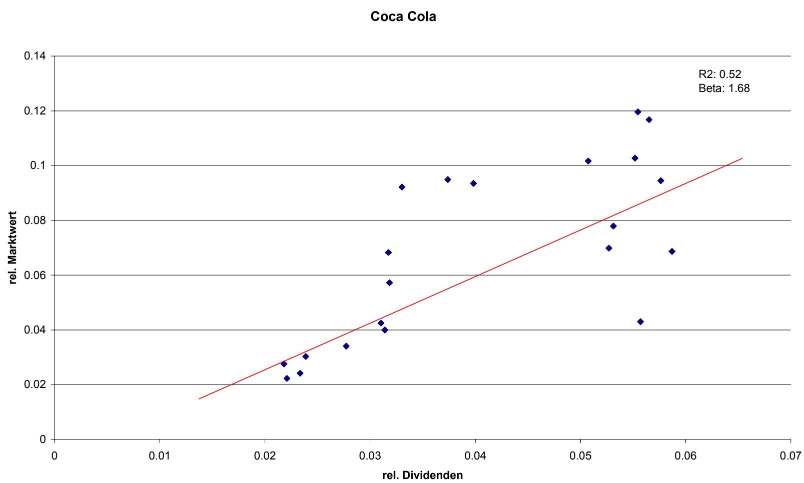
Relative MV follow relative DIV



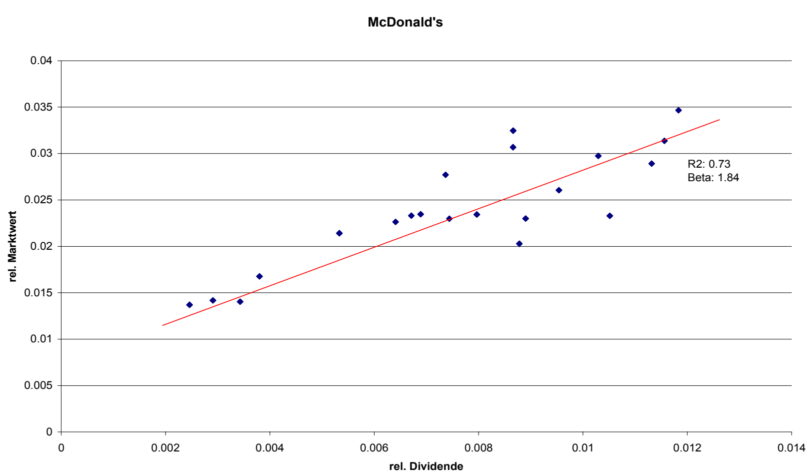
Relative MV follow relative DIV



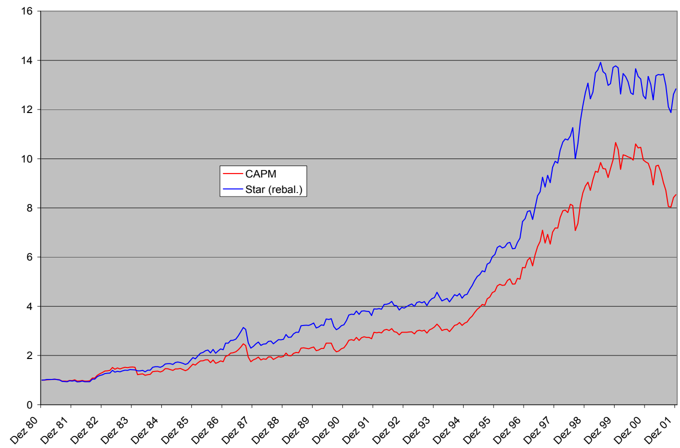
Relative MV follow relative DIV



Relative MV follow relative DIV

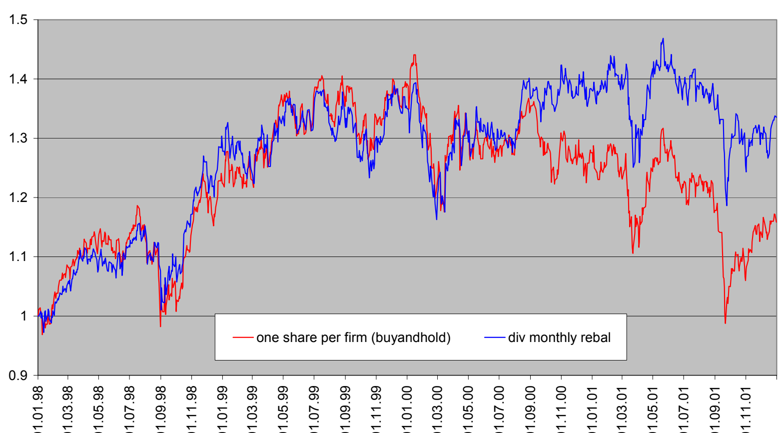


Evolutionary Investment Track Record (in sample)



In 20 years 50% outperformance if expectations of relative dividends are correct.

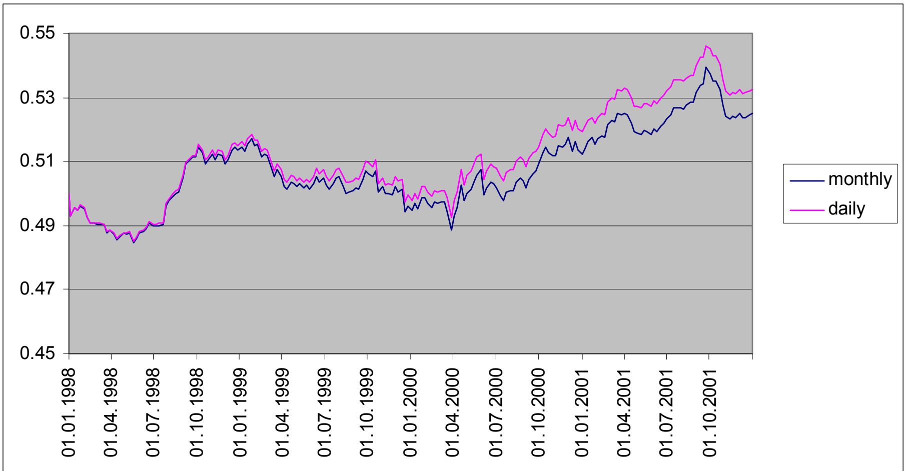
Evolutionary Investment (out of sample): DIV 93-97 Rebalance 98-2002



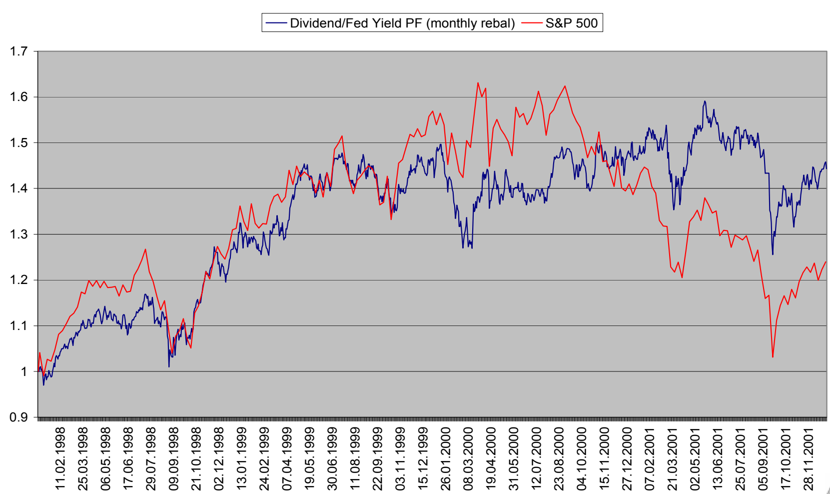
In 4 years outperformance of 13% over benchmark DJIA.

Evolutionary Investment Track Record (out of sample)

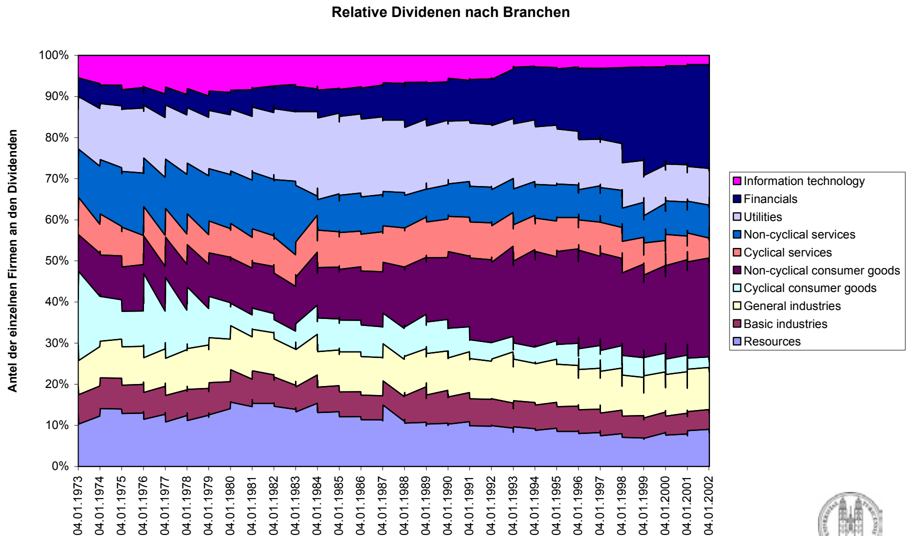
Monthly or daily rebalancing? In 4 years daily outperforms monthly by 1,4%.



Backtesting with Div-yield for Timing



Stationary Relative Dividends?



Producing GARCH

- Population Dynamics

Brock-Hommes (1998)

Two types of mean-variance traders:

rational expectations

adaptive expectations

Interplay of these groups can generate realistic asset prices with

Persistent deviations from fundamantel values and

Stochastic volatility and clustered volatility.

Illustration of Brock-Hommes (1998)

Strategies bs or sb:

momentum:	$P_{t-1} > P_{t-2}$	bs
	$P_{t-1} < P_{t-2}$	sb
mean-reverter:	$P_{t-1} > m$	sb
	$P_{t-1} < m$	bs

Price process: $P_t = P_{t-1} + \# b_t - \# s_t$

Lux (1998)

momentum:	$P_{t-1} > P_{t-2}$	bs
	$P_{t-1} < P_{t-2}$	sb
mean-reverter:	$P_{t-1} > m$	sb
	$P_{t-1} < m$	bs
outsiders:	Pt-1 > Pt-2>Pt-3	bs
	Pt-1 < Pt-2<Pt-3	sb

Price process: $P_t = P_{t-1} + \# b_t - \# s_t$

Population Dynamics: Imitate if Better

Lux (1998)

Two types of mean-variance traders:

rational expectations

adaptive expectations

Population Dynamics: „Imitate if better“

Interplay of these groups can generate realistic asset prices with

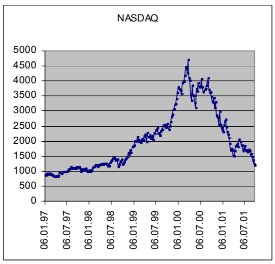
Persistent deviations from fundamantel values and

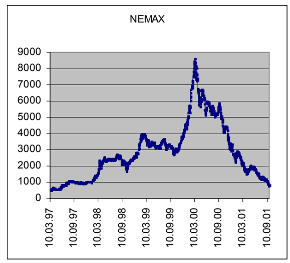
Stochastic volatility and clustered volatility.

Speculative bubbles in dtock markets

Stages of a bubble:

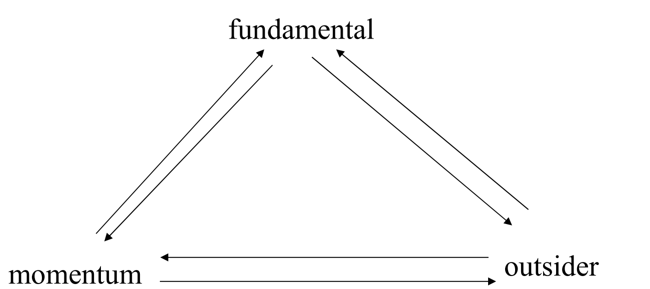
- Displacement (diffusion of new technology starts)
- Take off (stock prices show abnormal increase)
- Exuberance (stock prices grow at very high rate)
- Critical Stage (stock price growth slows down)
- Crash (stock prices start tumbling)





Population dynamics to explain bubbles

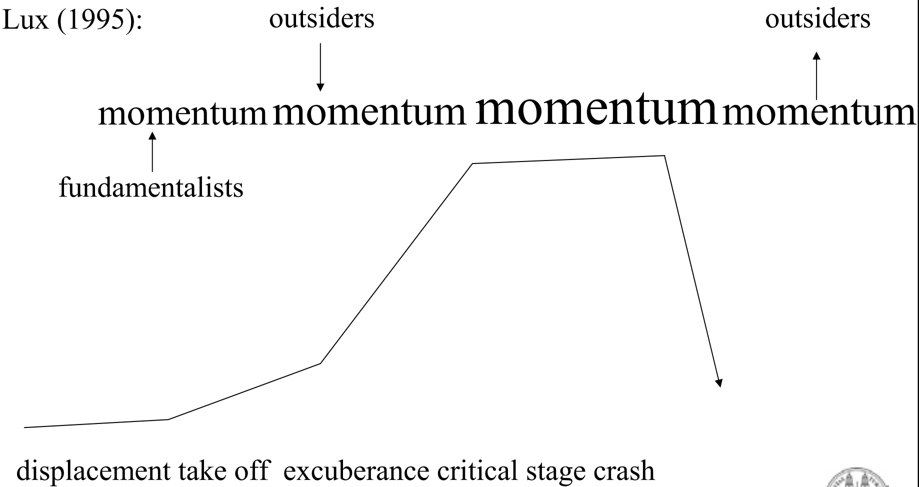
Lux (1995): Population dynamics



Change of strategy according to profitability: **„Imitate if better“**

Population dynamics to explain bubbles

Lux (1995):



Show Urs Trinkner Exel

LeBaron (2002)

Several types of traders with different sample period (memory):

short run

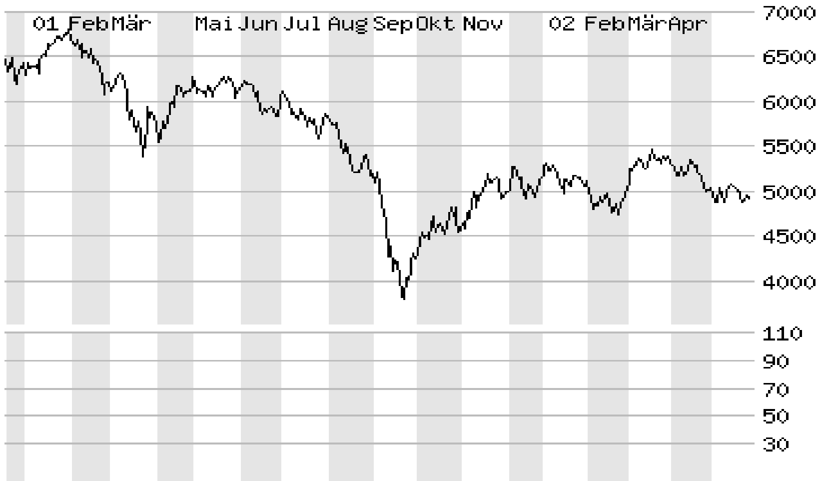
medium run

long run

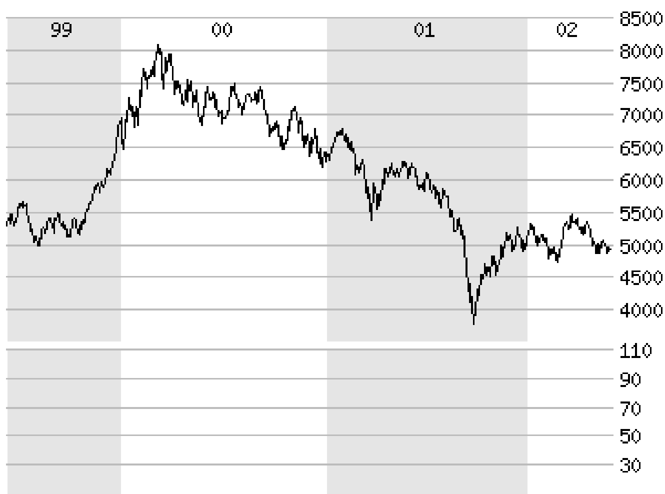
Populations dynamics: „imitate if better".

Sample Period: Intraday

Sample Period: 1 Year



Sample Period: 3 Years



Stochastic Volatility LeBaron (2002)

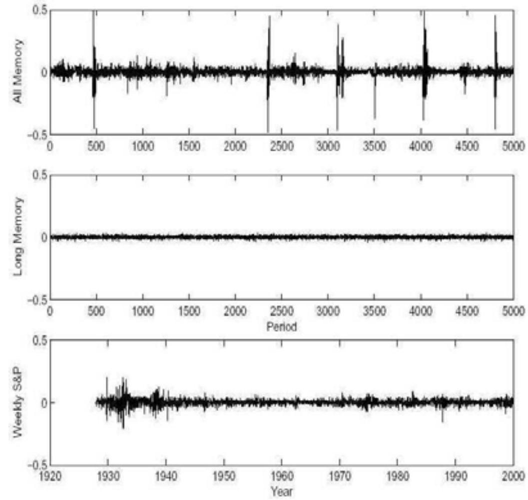
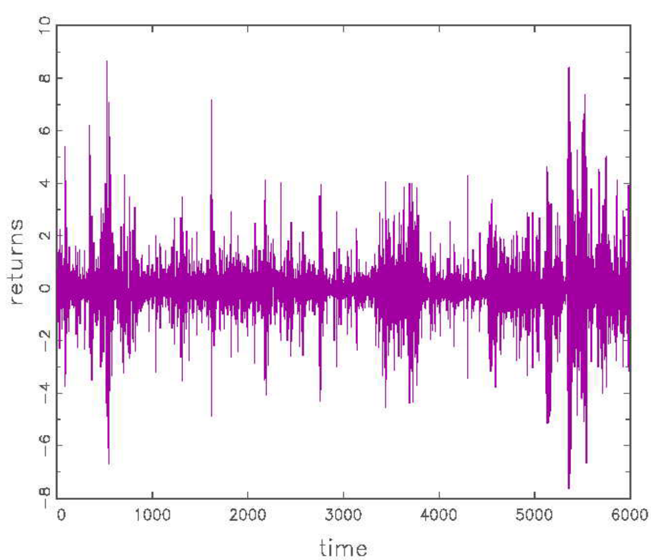


Figure 6: **Weekly Return Time Series** All memory, long memory, and S&P

Stochastic Volatility Lux (2002)



Summary Evolutionary Asset Pricing

LeBaron, Arthur, Palmer (99), Brock, Hommes (98), Lux (95)

heterogeneous agents:

λ^i rational and adaptive behavior

- genetic algorithm
- adaptive expectations
- sample period
- social learning

simulations:

asset prices show

- excess volatility
- momentum
- reversal

Conclusion: Main Features of Financial Markets

- Noise
- Anticipation Principle
- Reflexivity
- Beauty Contest
- Minority Game
- (Bounded) Rationality
- Market Selection