University of Zurich
Institute for Empirical Research in Economics

Advanced Portfolio Theory

NHH-Bergen

August 27th to September 9th 2003

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2. Foundations From Portfolio Theory

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Markowitz (52): MV-principle

„Various reasons recommend the use of expected return-variance of return rule, both as a hypothesis to explain well-established investment behavior and as a maxim to guide one's own action."

Markowitz, H.M. (1952) : "Portfolio Selection", *Journal of Finance*, 7, 77-91.

Jagannathan and Wang (1996)

"Mean-variance analysis and the Sharpe-Lintner-Mossin CAPM are widely viewed as the major contributions of academic research in the post-war era"

Jagannathan and Wang (1996): *The Conditional CAPM and the Cross-Sections of Expected Returns, Journal of Finance (51), 3--53.*

Campbell and Viciera (2002) on MV

„Most MBA courses, for example, still teach mean-variance analysis as if it were a universally accepted framework for portfolio choice"

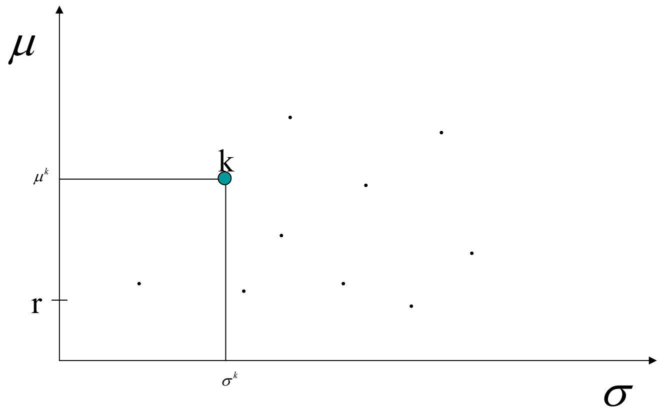
Campbell and Viciera (2002): Strategic Asset Allocation, OUP, page 7.

a) Mean-Variance Analysis

k=1,…,K assets

R^k gross return, $\mu^k=E(R^k)$ expected return,

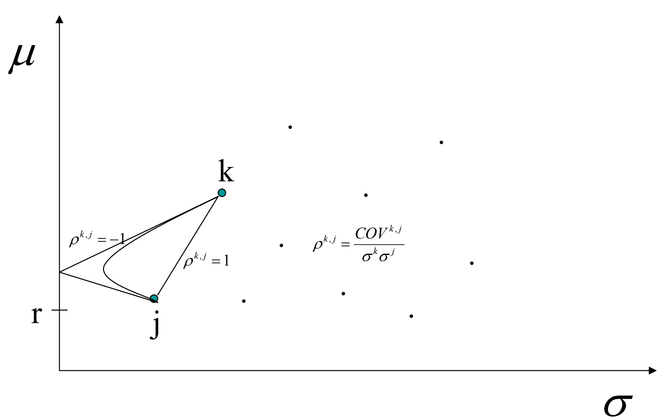
$\sigma^k=VAR(R^k)$ variance of return



a) Mean-Variance Analysis

Combining two assets k,j by portfolio $0 \le \lambda \le 1$

$\mu_\lambda = \lambda\mu^k + (1-\lambda)\mu^j$ and $\sigma_\lambda^2 = \lambda^2\sigma^k + (1-\lambda)^2\sigma^j + 2\lambda(1-\lambda)COV^{k,j}$



a) Mean-Variance Analysis

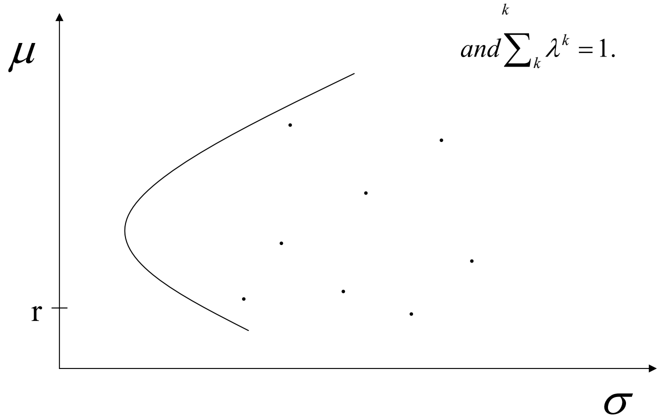
Combining all assets:

Efficient Frontier

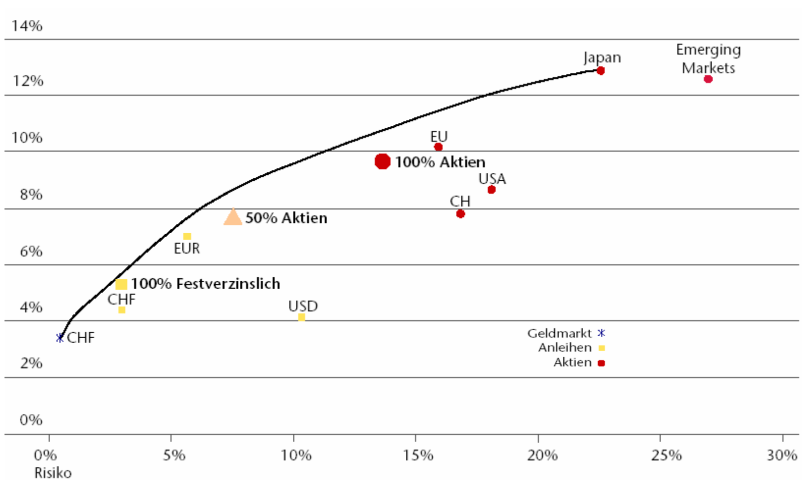
$$\min \sum_k \sum_j \lambda^{k,j} COV^{k,j}$$

$$s.t. \sum_k \lambda^k \mu^k = const$$

$$and \sum_k \lambda^k = 1.$$



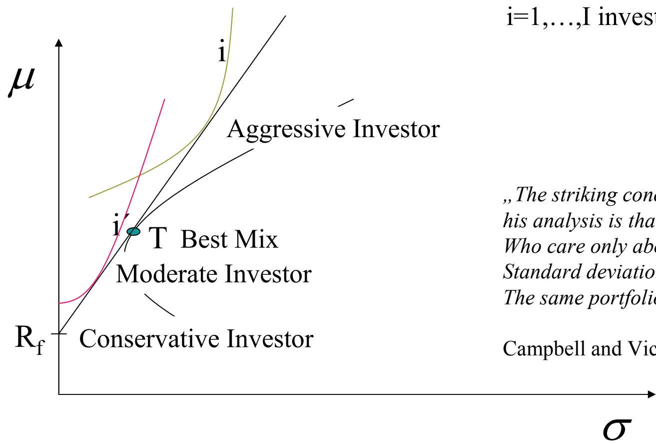
The Efficient Frontier



b) Two Fund Separation

$$\lambda^i = (\lambda_0^i, (1-\lambda_0^i)\lambda^T), i = 1,\ldots I$$

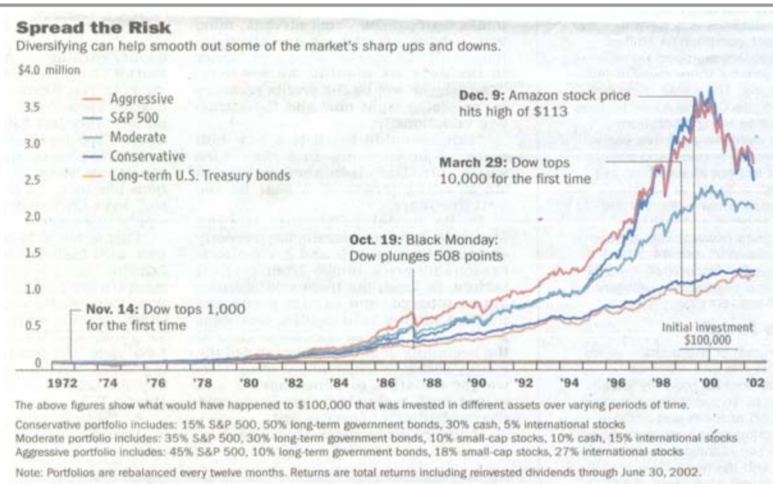
i=1,…,I investors



„The striking conclusion of his analysis is that all Investors Who care only about mean and Standard deviation will hold The same portfolio of risky assets."

Campbell and Viceira (2002), p.3.

An Asset Allocation Puzzle



Canner, Mankiw und Weil (1997) : An asset allocation puzzle, AER(87), pp.181-191.

An Asset Allocation Puzzle

Advisor In investor type	% of portfolio Cash	 Bonds	 Stocks	Ratio of bonds to stocks
(a) Fidelity				
Conservative	50	30	20	1.50
Moderate	20	40	40	1.00
Aggressive	5	30	65	0.46
(b) Merrill Lynch				
Conservative	20	35	45	0.78
Moderate	5	40	55	0.73
Aggressive	5	20	75	0.27
(c) New York Times				
Conservative	20	40	40	1.00
Moderate	10	30	60	0.50
Aggressive	0	20	80	0.25

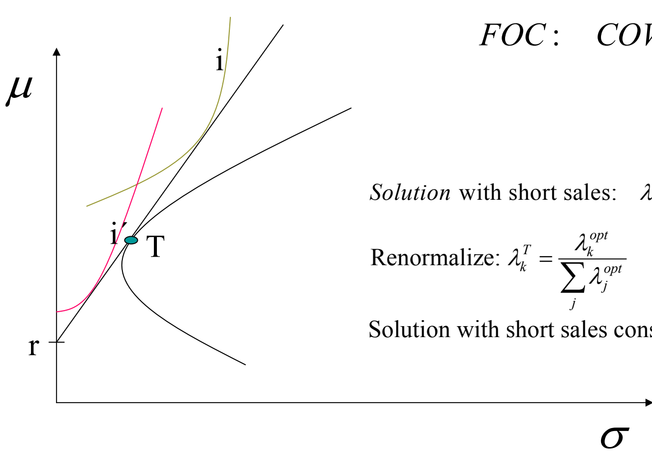
Canner, Mankiw und Weil (1997) : An asset allocation puzzle, AER(87), pp.181-191.

c) Solving for the optimal Portfolio

Quadratic Programming:

$$\max(\mu - R_f)\lambda_1 + \frac{\gamma^i}{2}\lambda_1' COV \lambda_1$$

$$FOC: \quad COV \lambda_1 = \frac{1}{\gamma^i}(\mu - R_f)$$



Solution with short sales: $\lambda_1 = \frac{1}{\gamma^i} COV^{-1}(\mu - R_f)$

Renormalize: $\lambda_k^T = \frac{\lambda_k^{opt}}{\sum_j \lambda_j^{opt}}$

Solution with short sales constraints: Gauss Algorithm!

Importance of means

Errors in Means, Variances and Covariances

Kallberg-Ziemba 1984, Chopra-Ziemba JPM 1993

observed mean

Replace true mean μ_i by $\mu_i(1+kZ_i)$ $\quad Z_i \sim N(0,1)$

size of error ⟶ scale factor k = 0.05 to 0.20

Replace true variances σ_i^2 by $\sigma_i^2(1+kZ_i)$

covariances σ_{ij}^2 by $\sigma_{ij}^2(1+kZ_i)$

10 DJIA Securities 1980-89 Alcoa, Boeing, Coke, Dupont, Sears, etc

Monthly data

$u(CE)=E_\xi u(\xi' x) \Rightarrow CE=u^{-1}[E_\xi(\xi' x)]$

Certainty equivalent $CE = u^{-1}$ (expected utility of risky portfolio)

Measure CE Loss u = exponential, normal distributions → mean-variance

exact formulas to compute $\mu - \frac{1}{t}\sigma^2$

Certainty equivalent loss $\Rightarrow CEL = \left(\frac{CE_{opt} - CE_{approx}}{CE_{opt}}\right)100$

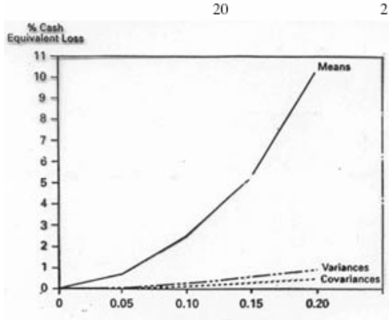
Mean Percentage Cash Equivalent Loss Due to Errors in Inputs

Average Ratio of CEL for Errors in Means, Variances and Covariances

t Risk Tolerance	Errors in Means vs Covariances	Errors in Means vs Variances	Errors in Variances vs Covariances
25	5.38	3.22	1.67
50	22.50	10.98	2.05
75	56.84	21.42	2.68
	↓	↓	↓
	20	10	2

The error depends on the risk tolerance but roughly

Error Mean	Error Var	Error Covar
20	2	1



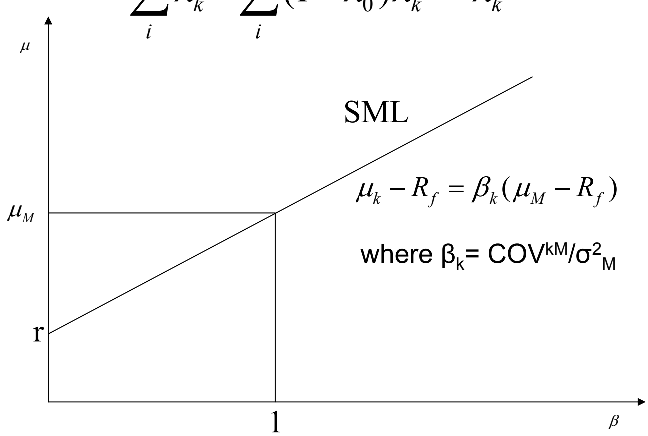
Conclusion: spend your money getting good estimates for means and use historical variances and covariances

Mean-Variance Tage vorführen

d) Capital Asset Pricing Model

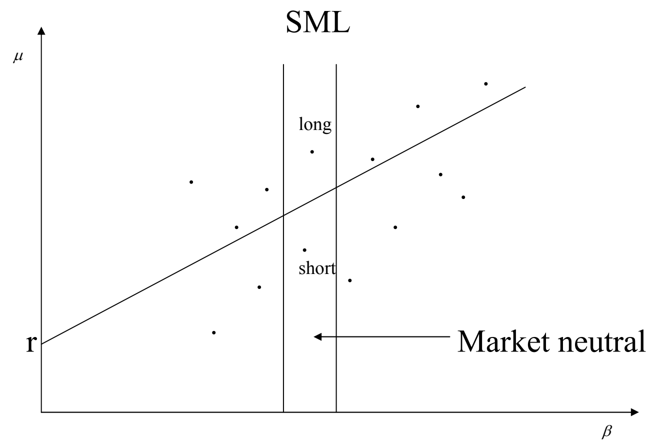
Equilibrium implications of mean-variance analysis:

$$\sum_i \lambda_k^i = \sum_i (1-\lambda_0^i)\lambda_k^T \; = \lambda_k^M$$



e) Hedge Fund Application

Long/Short Equity:



f) Issues left open by MV-Principle

- Is the MV-Rule consitent with rational portfolio choice?
- Is MV Behaviour a good description of what investors do?
- Does the CAPM hold with heterogenous beliefs or background risks.
- What does the MV Rule recommend if the market portfolio is not efficient and the tangential portfolio is underdiversified?
- Is MV Behaviour dynamically consistent?
- Is following the MV Rule a good risk management?