



University of Zurich  
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# Advanced Portfolio Theory

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IEW

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## 4. Rational Choice

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- A Model of Rational Choice
- The Meaning of Rationality
- Expected Utility Theory
- Expected Utility Theory in Action
- A Second Look at Mean-Variance
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### a) A Model of Rational Choice

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- Rational Search for Information
- Rational Reasoning
- Rational Framing of Alternatives
- Rational Selection of Best Alternative



Keeney (1982): Decision Analysis: An Overview, Operations Research.  
Howard (1988): Decision Analysis: Practice and Promise, Management Science.



## Howard (1988) A good decision

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„A good outcome is a future state of the world that we prize relative to other outcomes. A good decision is an action we take that is logically consistent with alternativee we percieve, the information we have, and the preferences we feel.“ (page 682).

R.A. Howard: “Decision Analysis: Practice and Promise“, Management Science (34), June 1988, p.679-695.



## Howard (1988) Typical Mistakes

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- Violations of the Independence Axiom
- Dependence on irrelevant alternatives
- Negative Value of information
- Violations of Stochastic Dominance
- Intransitive Preferences
- Dependence on sunk costs (regret).



## i. Rational Search For Information

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### Optimal Stopping:

Utility **loss** from cost of searching = expected utility **gain** from information

#### Example:

**LOSS** = Utility(wealth) – Utility(wealth – cost)

Task is to guess the mid point of an unknown interval in  $[0,100]$ .

**GAIN** =  $-|M - m|^2$ , where M is your guess and m is the true midpoint.

Purchase Signals  $x_i$  uniformly distributed in unknown interval.

Weak Law of Large Numbers:  $P\left(\left|\frac{1}{N}(x_1 + x_2 + \dots + x_N) - E(x)\right| \geq \varepsilon\right) \leq \frac{\sigma}{\varepsilon^2 N}$

Stigler, G.J. (1961): „The Economics of Information“, Journal of Political Economy 95, 213-225.



## ii. Rational Reasoning

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- Examples of Irrational Reasoning

  - Example of Howard (1988)

  - Example of Shefrin (2000)

- Rational Reasoning

  - Rational Framing

  - Rational Selection of the Best Alternative



## Irrational Reasoning (1)

---

Die outcome	1	2	3	4	5	6
Scheme A	\$600	\$700	\$800	\$900	\$1000	\$500
Scheme B	\$500	\$600	\$700	\$800	\$900	\$1000

„I prefer scheme A to scheme B because in 5 out of 6 cases I get a higher payoff in A than in B.“



## Irrational Reasoning (1)

---

Die outcome	1	2	3	4	5	6
Scheme A	\$600	\$700	\$800	\$900	\$1000	\$500
Scheme B	\$500	\$600	\$700	\$800	\$900	\$1000

„I prefer scheme A to scheme B because in 5 out of 6 cases I get a higher payoff in A than in B.“

However A and B are exactly the same loteries.



## Irrational Reasoning (2)

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First choose

(A) A sure gain of £2400

or

(B) a 25% chance of a £10.000 gain and a 75% chance of winning nothing at all.

Then choose

(C) a sure loss of £7500

or

(D) a 75% chance of a £10.000 loss and a 25% chance of losing nothing at all.

Your payoff is the combination of the two lotteries you have chosen!



## Irrational Reasoning (2)

---

Typical choice:

(A) because the expected payoff of (B) is £2500, the extra £100 is not enough to tempt people into taking a chance.

(D) Because this avoids the chance of a loss.



## Irrational Reasoning (2)

---

Typical choice:

(A) because the expected payoff of (B) is £2500, the extra £100 is not enough to tempt people into taking a chance.

(D) Because this avoids the chance of a loss.

Combined payoffs:

(A) and (D)     $£2400 - £7500 = -£5100$

(B) and (C)     $£2500 - £7500 = -£5000$ .



## iii. Rational Reasoning

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- Consider the **future** consequences of your choice
- Separate your **choice** from your **luck** (nature)
- Quantify **probabilistic** statements about nature
- State end of period **wealth** for your actions

	boom	recession
stocks	10	5
bonds	5	7
	p	1-p



## iv. Rational Selection of Best Alternative

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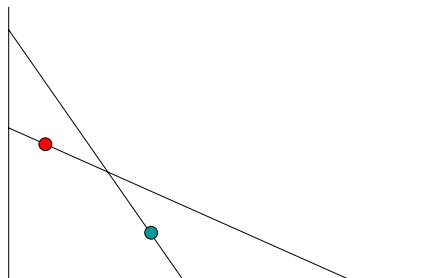
- Maximize Preference by choice of alternatives
- „Divide and conquer“
- Use quantitative procedures
- Interpret result carefully to get some intuition



## b) The Meaning of Rationality

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- Rationality of Choices?
  - Choose an action that maximizes your preference
  - Or at least avoid contradicting choices



## b) The Meaning of Rationality

---

- Rationality of Choices?
  - Choose an action that maximizes your preference
  - Or at least avoid contradicting choices
  
- Rationality of Preferences?
  - Base your Preferences on appealing principles (axioms)
  - Rank principles according to first order, second order etc.
  - Check consistency of these principles



(Marshak, (1951), “*Why ‘Should’ Statisticians and Businessmen Maximise ‘Moral Expectations’?*”)

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### **On the rationality of expected utility maximization:**

“The rule of maximizing the expectation of good on every occasion will, in some circumstances (see Marschak 1951), give you the greatest expectation of good in all.

But it only follows that rationality requires you to adopt this rule if rationality already says you should aim for the greatest expectation of good in all. And that is exactly what is in question.”



## My Point of View:

---

I understand Rationality as consistency with

### **Axiom 0:**

**More money is better than less money.**

I.e. behavior concerning risk and intertemporal asset allocation will not be called irrational if it does not contradict Axiom 0.



## Campbell and Viceira (2002)

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„ More specifically, we believe that any normative model should judge a portfolio by its total value, rather than by the values of the individual assets it contains, and should ultimately be based on the standard of living that the portfolio supports.“

Campbell and Viceira (2002): Strategic Asset Allocation, OUP, page 10.



## c) Expected Utility Theory

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- The Framework
- The Four Axioms
- The Representation Theorem
- Stochastic Dominance
- The Probability Triangle
- A second look at the MV-Principle
- Measures of Risk Aversion and their implication for portfolio choice
- Expected Utility Theory in Action
- Rational Probabilities
- Rational Time Preferences



## *(Kahneman and Tversky, Prospect Theory: An Analysis of Decision Under Risk, 1979)*

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“Expected Utility Theory has dominated the analysis of decision making under risk. It has been generally accepted as a normative model of rational choice [24], and widely applied as a descriptive model of economic behavior, e.g. [15,4].

Thus, it is assumed that all reasonable people would wish to obey the axioms of the theory [47,36], and that most people actually do, most of the time.



## i. The framework

---

Set of possible choices: X

X = e.g. set of Lotteries L

over a set of consequences C

p in L describes which consequence c in C is occurring with which probability

Example: C = { -100\$, 0\$, 100\$ }

p = (0.25, 0.25, 0.5)

Preference  $\succsim$  on L



## Deriving Lotteries from the State-Preference Model

---

Set of states  $S = \{1, \dots, S\}$

With probability measure  $(\text{prob}_1, \dots, \text{prob}_S) \gg 0$ .

Set of consequences  $C = \{c_1, \dots, c_i, \dots, c_n\}$ .

Derive Lotteries p on C by 
$$p_i = \sum_{\substack{s=1 \\ c_s=c_i}}^S \text{prob}_s$$



## ii. The Four Axioms

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- Completeness
- Transitivity
- Continuity
- Independence



### (1) Completeness

---

For all pair of lotteries  $p$  and  $q$ ,

We can either say  $p \succsim q$  *or*  $q \succsim p$ .

Is questionable for some set of consequences  $C$ ,

e.g.  $C = \{\text{die in car crash, die in plane crash}\}$

Seems o.k for monetary consequences.



## (2) Transitivity

---

For all lotteries  $p, q$  and  $r$ ,

If  $p \succcurlyeq q$  and  $q \succcurlyeq r$  then  $p \succcurlyeq r$ .

If there exists a market for  $p, q, r$  then violation of transitivity leads to a **money pump**, and hence to a violation of Axiom 0!



## (3) Continuity

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For all lotteries  $p, q$  and  $r$ ,

If  $p \succcurlyeq q \succcurlyeq r$  then there exists  $\alpha$  s. t.  $\alpha p + (1 - \alpha) r \sim q$ .

Is questionable for some set of consequences  $C$ ,

e.g.  $C = \{\text{die in car crash, get 1\$}, \text{get 1MIO \$}\}$

$p = (0, 1, 0)$ ,  $q = (0, 1, 0)$ ,  $r = (1, 0, 0)$ .

Seems o.k for pure monetary consequences.



## (4) Independence

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For all lotteries  $p, q, r$  and all  $0 \leq \alpha \leq 1$

If  $p \succsim q$  then  $\alpha p + (1 - \alpha)r \succsim \alpha q + (1 - \alpha)r$ .

Follows from Axiom 0 if lotteries are derived from state-preference model and markets are perfect and complete.

Is questionable otherwise.



## Independence Axiom Allais Paradox

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Urn with balls numbered  $0, 1, \dots, 99$ .

	0	1-10	11-99
A	50	50	50
B	0	250	50
A'			
B'			



## Independence Axiom Allais Paradox

---

Urn with balls numbered 0,1,...,99.

	0	1-10	11-99
A			
B			
A'	50	50	0
B'	0	250	0



## Independence Axiom Allais Paradox

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Urn with balls numbered 0,1,...,99.

	0	1-10	11-99
A	50	50	50
B	0	250	50
A'	50	50	0
B'	0	250	0

Typical choice but in contradiction to independence axiom:  $A \succ B$  and  $B' \succ A'$ .



## Independence Axiom Allais Paradox

---

Urn with balls numbered 0,1,...,99.

	0	1-10	11-99
A	50	50	50
B	0	250	50
A'	50	50	0
B'	0	250	0

Total payoff:      0                  0                  0

Arbitrage:

$A \succcurlyeq B$     Sell A to investor  
Buy B from him.

$B' \succcurlyeq A'$     Sell B' to investor  
Buy A' from him.

Win if preference is strict.



## Independence Axiom Allais Paradox

---

Lottery Approach:

	\$0	\$50	\$250
A	0%	100%	0%
B	1%	89%	10%
A'	89%	11%	0
B'	90%	0%	10%

Arbitrage:

$A \succcurlyeq B$     Sell A to investor  
Buy B from him.

$B' \succcurlyeq A'$     Sell B' to investor  
Buy A' from him.

Total payoff may be risky



# Independence Axiom Allais Paradox

Lottery Approach:

	\$0	\$50	\$250
A	0%	100%	0%
B	1%	89%	10%
A'	89%	11%	0
B'	90%	0%	10%

Note:

$$A = 0.11*(0.00, 1.00, 0.00) + 0.89*(0, 1, 0)$$

$$B = 0.11*(0.09, 0.00, 0.90) + 0.89*(0, 1, 0)$$

$$A' = 0.11*(0.00, 1.00, 0.00) + 0.89*(1, 0, 0)$$

$$B' = 0.11*(0.09, 0.00, 0.90) + 0.89*(1, 0, 0)$$

$$A \succcurlyeq B \quad B' \succcurlyeq A'$$

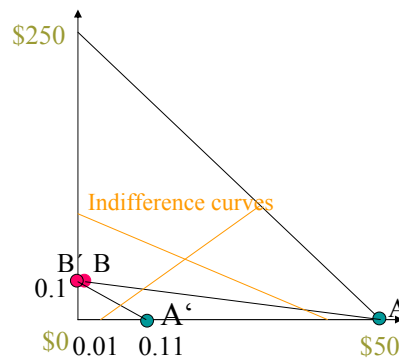
Contradicts Independence Axiom



# Independence Axiom Allais Paradox

Lottery Approach:

	\$0	\$50	\$250
A	0%	100%	0%
B	1%	89%	10%
A'	89%	11%	0
B'	90%	0%	10%



$$A \succcurlyeq B \quad B' \succcurlyeq A'$$

Contradicts parallel Indifferent Curves



## Independence Axiom

---

Suppose for the three states

$s_1 = \{0\}$ ,  $s_2 = \{1-10\}$ ,  $s_3 = \{11-99\}$ .

There are Arrow securities and that agents can transfer money between those states back and forth at given prices  $\pi_{s_i} / \pi_{s_j}$

Then Violations of Independence Axiom are Violations of Axiom 0



## Ellsberg Paradox

---

Urn with 90 balls 30 red 30 yellow or black in unknown proportion



	red	yellow	black
A	50\$	0	0
B	0	0	50\$
A'	50\$	50\$	0
B'	0	50\$	50\$

Typical choice but in contradiction to independence axiom:  $A \succcurlyeq B$  and  $B' \succcurlyeq A'$ .



## Arbitrage Ellsberg Paradox

Urn with 90 balls 30 red 30 yellow or black in unknown proportion

	red	yellow	black		Arbitrage:
A	50\$	0	0	-1	$A \succcurlyeq B$ Sell A to investor Buy B from him.
B	0	0	50\$	+1	
A'	50\$	50\$	0	+1	$B' \succcurlyeq A'$ Sell B' to investor Buy A' from him.
B'	0	50\$	50\$	-1	Win if strict preference.
Total Position	0	0	0		



## Independence Axiom Ellsberg Paradox

Since the proportion of yellow to black balls is **ambiguous** it is not likely to have markets unbundling the two. I.e. markets are **incomplete** as e.g. in

	red	yellow	black
k=1	1	0	0
k=2	0	1	1

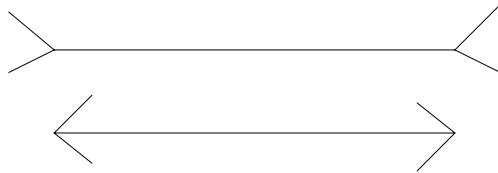
Hence typical choice is not irrational because it cannot be refuted by Axiom 0



## Arrow (1971) Optical Illusion

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“The temptation to avoid facing squarely the issue of conditional preference is cleverly exploited in constructing the examples. I feel strongly that they belong to the family of ‘optical illusions’.”



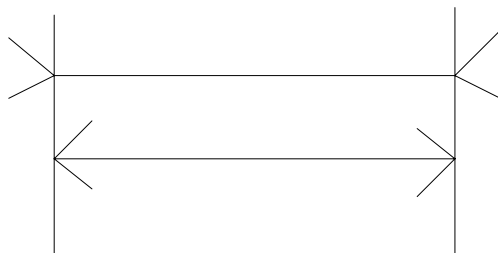
(Arrow, *Axiomatic Theories of Choice, Cardinal Utility and Subjective Probability: a Review*; Opening lecture for the Workshop in Economic Theory organised by the International Economic Association in Bergen in 1971).



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“The four assumptions just reviewed may be summarised by saying that a consistent decision-maker is assumed always to be able to compare (transitively) the attractiveness of acts, of hypothetical acts and of consequences as well as the likelihood of events. These requirements are minimal, in the sense that no consistency of behaviour may be expected if any one of them is violated.”



## Arrow: „Get Lost!“

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“In other words, a person who does not accept the axioms of simple ordering for conditional acts, consequences and events, should not expect any assistance from scientific methods in handling decision problems.”

(Arrow, *Axiomatic Theories of Choice, Cardinal Utility and Subjective Probability: a Review*; Opening lecture for the Workshop in Economic Theory organised by the International Economic Association in Bergen in 1971).





### iii. The Representation Theorem



Let  $\succsim$  be a preference order that is complete, transitive and continuous then  $\succsim$  is representably by an expected utility function, i.e.

$$p \succ q \Leftrightarrow E_u(p) > E_u(q)$$

if and only if  $\succsim$  satisfies the independence Axiom.

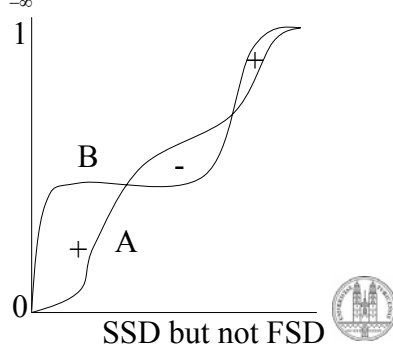
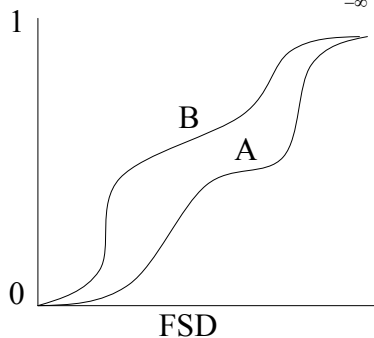
$$E_u(p) := \sum_{i=1}^n p_i u(c_i) \quad \text{separates beliefs from risk attitudes}$$



### iv. Stochastic Dominance

Distribution Function:  $A(y) = P(x \leq y)$

- A FSD B iff for all  $y$   $A(y) < B(y)$ .
- A FSD B iff for all  $y$   $\int_{-\infty}^y x dA(x) < \int_{-\infty}^y x dB(x)$



## Stochastic Dominance and Expected Utility

---

(1) A FSD B if and only if

for all  $u$  monotone  $Eu(A) > Eu(B)$ .

(2) A FSD B if and only if

for all  $u$  monotone and concave

$Eu(A) > Eu(B)$ .



## Axiom 0 and Stochastic Dominance

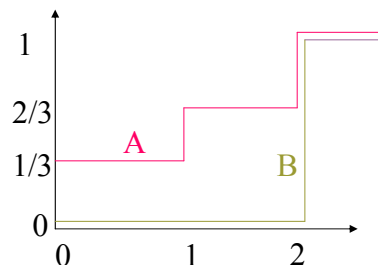
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Consider state-preference model

Axiom 0 = State-dominance:  $c^i > d^i \Rightarrow U^i(c^i) > U^i(d^i)$

State Dominance implies FSD but reverse is no true:

	1/3	1/3	1/3
	S=1	S=2	S=3
A	0	1	2
B	1	2	1



## A First Result on Diversification

---

Consider a collection of assets with identically distributed returns.

Then the optimal portfolio of every risk averse expected utility maximizer has equal weights for all assets.



## A First Result on Diversification

---

Normal Distribution:

Suppose returns are jointly normal  $\sim N(\mu, \sigma)$

Then  $E_u(x) = \int u(x) dN(x; \mu, \sigma) = \int u\left(\frac{x - \mu}{\sigma}\right) d\hat{N}(x) = V(\mu, \sigma)$

Consider  $\max_{\lambda_1} V((\mu - r)\lambda_1, \lambda_1 \text{COV} \lambda_1)$

*FOC*:  $\text{COV} \lambda_1 = -\frac{\partial_{\mu} V}{\partial \sigma^2} (\mu - r)$

Hence if identically distributed returns then

$\text{COV}_{j,k} = \sigma^2$  and  $\mu_j = \mu$ , all  $j, k$  hence  $\lambda_1$  is  $1/n$  portfolio



## A Result on Diversification

---

General Distributions:

Argue using SSD:

Consider  $R_{1/n} = \frac{1}{n} \sum_{k=1}^n R_k$  and  $R_\lambda = \sum_k \lambda_k R_k$  Let  $R_\Delta = R_{1/n} - R_\lambda$

For all  $\lambda \in \Delta^n$  we get  $E(R_\lambda) = \sum_k \lambda_k E(R_k) = \mu$

Consider  $R_\lambda = R_{1/n} + R_\Delta$

Note  $E(R_\Delta) = 0$  and  $E(R_\Delta | R_{1/n}) = \sum_k (\lambda_k - \frac{1}{n}) E(R_k | R_{1/n}) = 0$

Hence  $R_{1/n}$  SSD  $R_\lambda$  and every risk averse  $E_u$  chooses the portfolio  $1/n$ .



## A Result on Diversification

---

An example:

Two fair coins: If H win 1 if T loose 1.

		1/n	(1,0)	(0,1)	$\lambda$
H	H	1	1	1	1
	T	0	1	-1	$2\lambda - 1$
T	H	0	-1	1	$1 - 2\lambda$
	T	-1	-1	-1	-1

Note that

$$u(1 - 2\lambda) + u(2\lambda - 1) < 2u(0)$$

Hence by concavity of  $u$

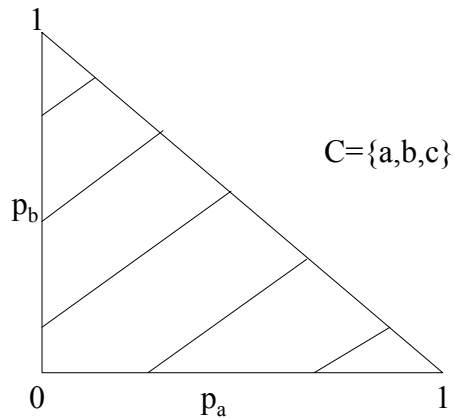
$$1/n \geq \lambda \text{ for any } \lambda$$



## v) The Probability Triangle

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In a probability triangle  $E_u(p) := \sum_{i=1}^n p_i u(c_i)$   
has parallel lines as indifference curves.



## vi. A Second Look at Mean-Variance

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Is the Mean-Variance Principle always rational?

- MV as a special case of expected utility
- Mean-Variance Paradox
- Violating the Independence Axiom



## Mean Variance Principle is a special case of expected utility

---

- Quadratic Utility:

$$E_u(x) = \sum_s p_s \left( x_s - \frac{\gamma}{2} x_s^2 \right) = \mu(x) - \frac{\gamma}{2} \mu(x^2) = \mu(x) \left( 1 - \frac{\gamma}{2} \mu(x) \right) - \frac{\gamma}{2} \sigma^2(x) = V(\mu, \sigma)$$

- Normally Distributed Returns:

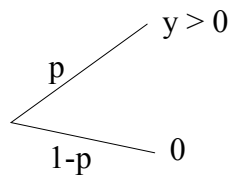
$$E_u(x) = \int u(x) dN(x; \mu, \sigma) = \int u\left(\frac{x - \mu}{\sigma}\right) d\hat{N}(x) = V(\mu, \sigma)$$



## Mean-Variance Paradox (violating Axiom 0)

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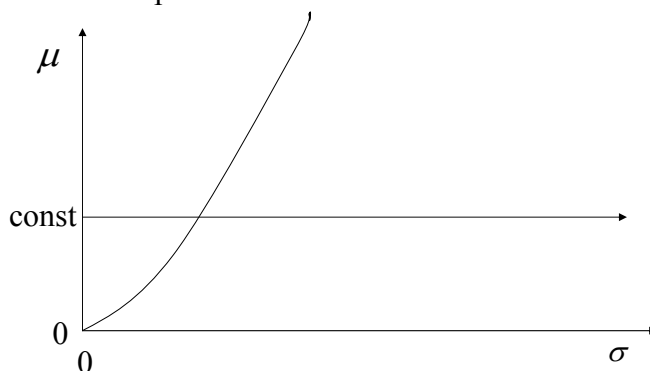
Lottery



Let  $y \rightarrow \infty$

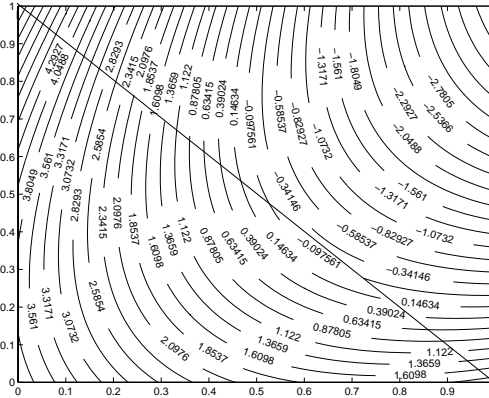
while  $p y = \text{const}$

Then  $\sigma \rightarrow \infty, \mu = \text{const}$



# Violating the Independence Axiom

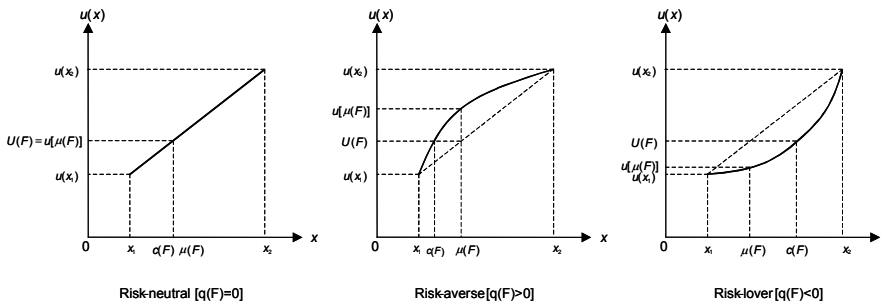
Let  $C=(2,4,6)$  and consider the indifference curves of  $\mu - \sigma^2$  in the probability triangle:



## vii. Measures of Risk Aversion

Certainty equivalent:  $u(c(F)) = \int_C u(x) dF(x) =: U(F)$

$q(F) = \mu(F) - c(F)$  where  $\mu(F) = \int x dF(x)$



## vii. Measures of Risk Aversion

---

- $ARA(x) = - \frac{u''(x)}{u'(x)}$   
determines asset allocation (units) when income changes
- $RRA(x) = ARA(x) \cdot x$   
determines asset allocation (shares) when income changes
- $RT(x) = 1/ARA(x)$   
determines two fund separation



## vii. Measures of Risk Aversion

---

The demand for risky assets **increases** with income iff  $ARA(x)$  **decreases** with income.

Examples:

$$u(x) = -e^{-\alpha x} \quad \text{has constant ARA}$$

$$u(x) = x - \frac{1}{\gamma} x^2 \quad \text{has increasing ARA}$$

$$u(x) = x^\alpha \quad \text{has decreasing ARA}$$



## vii. Measures of Risk Aversion

---

The wealth share of risky assets **increases** with income iff  $RRA(x)$  **decreases** with income.

Examples:

$$u(x) = x^\alpha \quad \text{has constant RRA}$$

$$u(x) = -e^{-\alpha x} \quad \text{has increasing RRA}$$



## vii. Measures of Risk Aversion

---

If  $RT^i(x) = \alpha^i + \beta x, i = 1, \dots, I$  (Linear Risk Tolerance)

then  $\lambda^i(x) = (\lambda_0^i(x), (1 - \lambda_0^i(x))\lambda^T(x))$ . (Two-Fund-Separation)

Class of LRT: (HARA-family)

$$u^i(x) = \begin{cases} \frac{(\alpha^i + \beta x)^{1 - \frac{1}{\beta}}}{\frac{1}{\beta}(1 - \frac{1}{\beta})} & \text{for } \beta \neq 0 \text{ and } \beta \neq 1 \\ -\alpha^i \exp(-\frac{x}{\alpha^i}) & \text{for } \beta = 0 \\ \ln(\alpha^i + x) & \text{for } \beta = 1 \end{cases}$$



## viii. Expected Utility Theory in Action

### Giving Advice with Eu-Theory

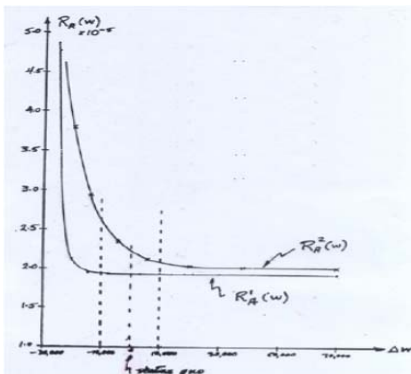
The client's input: Risk-preference  
(to be estimated by the advisor)

The advisor's input:

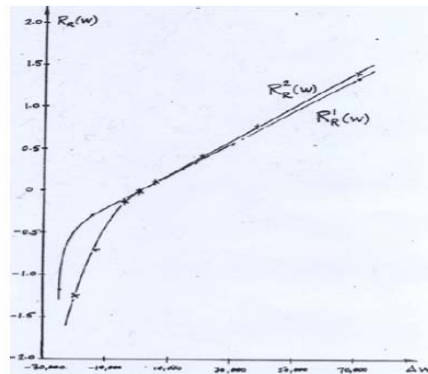
- Making aware of risks (risk consciousness)
- Checking Risk tolerance
- Balancing Risk Preference with Risk Tolerance.



## Estimating Utility Functions



(a) Absolute risk aversion function estimated by the certainly equivalent method (1) and gain and loss equivalent method (2)



(b) Relative risk aversion function estimated by the certainly equivalent method (1) and gain and loss equivalent method (2)



## viii. Expected Utility Theory in Action

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Case Study: Bank Leu Expert System

- Questionnaire
- Masterportfolios
- The matching process



## viii. Expected Utility Theory in Action

---

Case Study:

Bank Leu Questionnaire

- Risk Tolerance
- Risk Awareness
- Risk Preference
- Life Cycle Planning

Masterportfolios



Chancen und Risiken in den  
Finanzmärkten:  
Eine Analyse seit 1950

April 2003

Willkommen im Private Banking

Bank Leu  
1873



# Risk Awareness

## Question 1

\*: just one answer per question possible

Think about the most important asset in your portfolio you have invested in the last few years:

- |                                                         |                                                              |                                                                                                |
|---------------------------------------------------------|--------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| a) What return did you want to achieve with that asset? | b) How high did you estimate the maximal return at the time? | c) What would have been your estimation at the time of the highest possible loss of the asset? |
| Points*                                                 | Points*                                                      | Points*                                                                                        |
| Smaller than 4% <input type="checkbox"/> 10             | Smaller than 4% <input type="checkbox"/> 10                  | 0-6% <input type="checkbox"/> 10                                                               |
| 4-6% <input type="checkbox"/> 20                        | 4-6% <input type="checkbox"/> 20                             | 6-15% <input type="checkbox"/> 20                                                              |
| 6-8% <input type="checkbox"/> 30                        | 6-8% <input type="checkbox"/> 30                             | 15-25% <input type="checkbox"/> 30                                                             |
| 8-10% <input type="checkbox"/> 40                       | 8-10% <input type="checkbox"/> 40                            | 25-40% <input type="checkbox"/> 40                                                             |
| Larger than 10% <input type="checkbox"/> 50             | Larger than 10% <input type="checkbox"/> 50                  | More than 40% <input type="checkbox"/> 50                                                      |

Smallest number of points of all answers:

Please copy the value into the blue box on the reporting sheet



# Risk Preference

## Question 2

\*: just one answer per question possible

You would like to invest CHF 1'000'000. With the investment you have a 50% chance to gain CHF 1'000'000 within one year and a 50% to have less than CHF 1'000'000. How much of the CHF 1'000'000 should be available in the worst case so that you still invest?

- |                                              |                                               |
|----------------------------------------------|-----------------------------------------------|
| Points*                                      | Points*                                       |
| 1'000'000 <input type="checkbox"/> 0         | At least 750'000 <input type="checkbox"/> 30  |
| At least 940'000 <input type="checkbox"/> 10 | At least 600'000 <input type="checkbox"/> 40  |
| At least 850'000 <input type="checkbox"/> 20 | Less than 600'000 <input type="checkbox"/> 50 |

Number of points of the answer to question 2:

Please copy the value into the red box on the reporting sheet



# Risk Tolerance

## Question 3

*Possibly you would like to have the investment to be at your disposal at a certain point in the future for a purchase or would like it back.*

- When do you plan or estimate that you would like the investment to be at your disposal?

Surely in...	Points	Probably in...	Points
Below 3 years <input type="checkbox"/>	40	Below 3 years <input type="checkbox"/>	30
3 years <input type="checkbox"/>	30	3 years <input type="checkbox"/>	20
4 years <input type="checkbox"/>	20	4 years <input type="checkbox"/>	10
5 years <input type="checkbox"/>	10	5 years <input type="checkbox"/>	5
6 years <input type="checkbox"/>	5	6 years <input type="checkbox"/>	0
8 years and more <input type="checkbox"/>	0	8 years and more <input type="checkbox"/>	0

- b) I will need from amount invested

	Factor
One quarter <input type="checkbox"/>	$\frac{1}{4}$
The half <input type="checkbox"/>	$\frac{1}{2}$
Three quarters <input type="checkbox"/>	$\frac{3}{4}$
Everything <input type="checkbox"/>	1

Sum of points of the given answers to question 3a:

Please multiply the sum of points from question 3a) with the factor from question 3b):

Please copy the value into the **dark blue** box on the reporting sheet



# Risk Tolerance

## Question 4

\*: several answers per question possible

*During a life things often happen that cannot be arranged for exactly. In those situations financial resources are sometimes needed.*

- a) In which of the following events would you have to access the invested capital?

	Points		Points
Buying of a house under favourable conditions <input type="checkbox"/>	10	Foundation of an own company <input type="checkbox"/>	10
Rise of mortgage rates <input type="checkbox"/>	10	Invalidity <input type="checkbox"/>	10
Marriage or divorce <input type="checkbox"/>	10	In case of a severe illness <input type="checkbox"/>	10
In case of your death the beneficiary the investment would have to be at their disposal <input type="checkbox"/>	10	Temporary loss of your job for a half of year <input type="checkbox"/>	10
Education of children <input type="checkbox"/>	10	None of all the cases will occur <input type="checkbox"/>	0
Premature retirement <input type="checkbox"/>	10	For none of the cases I require any of invested amount <input type="checkbox"/>	0

Sum of all answers to question 4a):

- b) Then I need from the amount invested

	Factor
One quarter <input type="checkbox"/>	$\frac{1}{4}$
The half <input type="checkbox"/>	$\frac{1}{2}$
Three quarters <input type="checkbox"/>	$\frac{3}{4}$
Everything <input type="checkbox"/>	1

Please multiply the sum of points from question 4a) with the factor from question 4b):

Please copy the value into the **violet** box on the reporting sheet



# Risk Tolerance

## Question 5

*In everyday life problems can occur from which you possibly would like to have the amount invested to be at your disposal.*

a) In which of the following events would like to have the amount invested to be at your disposal?

- |                                                                         | Points                      |
|-------------------------------------------------------------------------|-----------------------------|
| Car accident without the insurance paying and you have to buy a new car | <input type="checkbox"/> 50 |
| You purchase something expensive for your house, such as furniture      | <input type="checkbox"/> 50 |
| Temporary loss of your job for a month                                  | <input type="checkbox"/> 50 |
| None of all the cases will occur                                        | <input type="checkbox"/> 0  |
| For none of the cases I require any of invested amount                  | <input type="checkbox"/> 0  |

Sum of all the answers to question 5a):

b) Then I need from the amount invested

- |                | Factor                                 |
|----------------|----------------------------------------|
| One quarter    | <input type="checkbox"/> $\frac{1}{4}$ |
| The half       | <input type="checkbox"/> $\frac{1}{2}$ |
| Three quarters | <input type="checkbox"/> $\frac{3}{4}$ |
| Everything     | <input type="checkbox"/> 1             |

Please multiply the sum of points from question 5a) with the factor from question 5b):

Please copy the value into the pink box on the reporting sheet



# Life Cycle Planning

## Question 6

*To include the amount to invest in an overall wealth plan we require a bit of general information from you.*

- |                                                                                                                                                                                                                                                                                                                                                                                                                        |                          |                          |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|--------------------------|----|---------------------|--------------------------|-----|-------------|--------------------------|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|--------------------------|----|----------------|--------------------------|-----|---------------|--------------------------|-----|----------|--------------------------|---|-----------|--------------------------|----|
| <p>a) Your age in years:</p> <table border="0"> <tr> <td>Below 30</td> <td><input type="checkbox"/></td> <td>0</td> </tr> <tr> <td>30-60</td> <td><input type="checkbox"/></td> <td>10</td> </tr> <tr> <td>Above 60</td> <td><input type="checkbox"/></td> <td>0</td> </tr> </table>                                                                                                                                   | Below 30                 | <input type="checkbox"/> | 0  | 30-60               | <input type="checkbox"/> | 10  | Above 60    | <input type="checkbox"/> | 0   | <p>b) You are:</p> <table border="0"> <tr> <td>Self-employed</td> <td><input type="checkbox"/></td> <td>0</td> </tr> <tr> <td>Retired</td> <td><input type="checkbox"/></td> <td>10</td> </tr> <tr> <td>Worker</td> <td><input type="checkbox"/></td> <td>0</td> </tr> <tr> <td>Employed</td> <td><input type="checkbox"/></td> <td>0</td> </tr> <tr> <td>Executive</td> <td><input type="checkbox"/></td> <td>10</td> </tr> </table> | Self-employed | <input type="checkbox"/> | 0  | Retired        | <input type="checkbox"/> | 10  | Worker        | <input type="checkbox"/> | 0   | Employed | <input type="checkbox"/> | 0 | Executive | <input type="checkbox"/> | 10 |
| Below 30                                                                                                                                                                                                                                                                                                                                                                                                               | <input type="checkbox"/> | 0                        |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| 30-60                                                                                                                                                                                                                                                                                                                                                                                                                  | <input type="checkbox"/> | 10                       |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Above 60                                                                                                                                                                                                                                                                                                                                                                                                               | <input type="checkbox"/> | 0                        |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Self-employed                                                                                                                                                                                                                                                                                                                                                                                                          | <input type="checkbox"/> | 0                        |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Retired                                                                                                                                                                                                                                                                                                                                                                                                                | <input type="checkbox"/> | 10                       |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Worker                                                                                                                                                                                                                                                                                                                                                                                                                 | <input type="checkbox"/> | 0                        |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Employed                                                                                                                                                                                                                                                                                                                                                                                                               | <input type="checkbox"/> | 0                        |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Executive                                                                                                                                                                                                                                                                                                                                                                                                              | <input type="checkbox"/> | 10                       |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| <p>c) Number of children (Small children or children in education)</p> <table border="0"> <tr> <td>0</td> <td><input type="checkbox"/></td> <td>0</td> </tr> <tr> <td>1-3</td> <td><input type="checkbox"/></td> <td>-10</td> </tr> <tr> <td>more than 3</td> <td><input type="checkbox"/></td> <td>-20</td> </tr> </table>                                                                                            | 0                        | <input type="checkbox"/> | 0  | 1-3                 | <input type="checkbox"/> | -10 | more than 3 | <input type="checkbox"/> | -20 | <p>d) Do you own real estate?</p> <table border="0"> <tr> <td>Yes</td> <td><input type="checkbox"/></td> <td>20</td> </tr> <tr> <td>No</td> <td><input type="checkbox"/></td> <td>0</td> </tr> </table>                                                                                                                                                                                                                               | Yes           | <input type="checkbox"/> | 20 | No             | <input type="checkbox"/> | 0   |               |                          |     |          |                          |   |           |                          |    |
| 0                                                                                                                                                                                                                                                                                                                                                                                                                      | <input type="checkbox"/> | 0                        |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| 1-3                                                                                                                                                                                                                                                                                                                                                                                                                    | <input type="checkbox"/> | -10                      |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| more than 3                                                                                                                                                                                                                                                                                                                                                                                                            | <input type="checkbox"/> | -20                      |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Yes                                                                                                                                                                                                                                                                                                                                                                                                                    | <input type="checkbox"/> | 20                       |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| No                                                                                                                                                                                                                                                                                                                                                                                                                     | <input type="checkbox"/> | 0                        |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| <p>e) What part of your wealth (real estate, capital investments, and similar) does the amount to invest approximately account for?</p> <table border="0"> <tr> <td>Less than 10%</td> <td><input type="checkbox"/></td> <td>20</td> </tr> <tr> <td>Between 10% and 20%</td> <td><input type="checkbox"/></td> <td>10</td> </tr> <tr> <td>20% or more</td> <td><input type="checkbox"/></td> <td>0</td> </tr> </table> | Less than 10%            | <input type="checkbox"/> | 20 | Between 10% and 20% | <input type="checkbox"/> | 10  | 20% or more | <input type="checkbox"/> | 0   | <p>f) What part of your annual income does the amount to invest account for?</p> <table border="0"> <tr> <td>Less than 50%</td> <td><input type="checkbox"/></td> <td></td> </tr> <tr> <td>Between 50-80%</td> <td><input type="checkbox"/></td> <td>-10</td> </tr> <tr> <td>More than 80%</td> <td><input type="checkbox"/></td> <td>-15</td> </tr> </table>                                                                         | Less than 50% | <input type="checkbox"/> |    | Between 50-80% | <input type="checkbox"/> | -10 | More than 80% | <input type="checkbox"/> | -15 |          |                          |   |           |                          |    |
| Less than 10%                                                                                                                                                                                                                                                                                                                                                                                                          | <input type="checkbox"/> | 20                       |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Between 10% and 20%                                                                                                                                                                                                                                                                                                                                                                                                    | <input type="checkbox"/> | 10                       |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| 20% or more                                                                                                                                                                                                                                                                                                                                                                                                            | <input type="checkbox"/> | 0                        |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Less than 50%                                                                                                                                                                                                                                                                                                                                                                                                          | <input type="checkbox"/> |                          |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| Between 50-80%                                                                                                                                                                                                                                                                                                                                                                                                         | <input type="checkbox"/> | -10                      |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |
| More than 80%                                                                                                                                                                                                                                                                                                                                                                                                          | <input type="checkbox"/> | -15                      |    |                     |                          |     |             |                          |     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                          |    |                |                          |     |               |                          |     |          |                          |   |           |                          |    |

Sum of all answers to question 6:

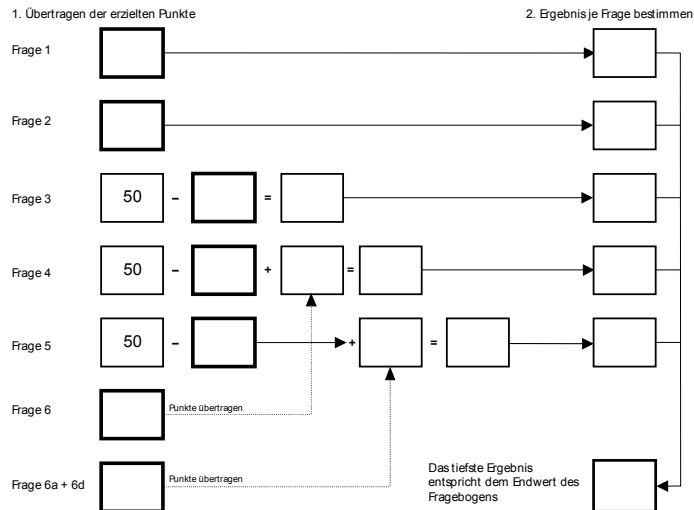
Please copy the value into the green box on the reporting sheet

Sum of all answers to the questions from 6a) until 6d):

Please copy the value into the light green box on the reporting sheet



# Calculation of Result



# The recommended portfolios

Punkte	Höchstens 8	8 bis 18	18 bis 28	28 bis 40	mehr als 40
	«100% Festverzinslich»	«25%Aktien»	«50% Aktien»	«75% Aktien»	«100% Aktien»
Aufteilung nach Anlagekategorien in %					
Liquidität CHF	5	5	5	5	5
Anleihen CHF	60	48	29	12	
Anleihen EUR	35	22	16	8	
Aktien Schweiz		10	19	27	33
Aktien Europa		7	15	23	28
Aktien USA		6	11	17	22
Aktien Japan		2	4	6	8
Aktien Emerging Markets			1	2	4



## ix. Rational Probabilities

---

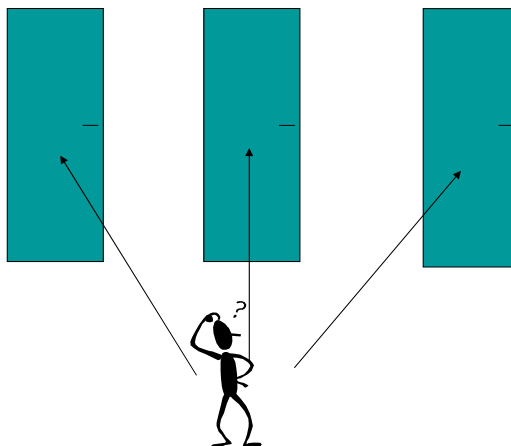
- Joint Probabilities:  $P(A \text{ and } B) \leq P(A)$
- Conditional Prob:  $P(A|B) = P(A \text{ and } B)/P(B)$
- Independence:  $P(A \text{ and } B) = P(A) * P(B)$
- Updating of Probabilities: Bayes Law

$$p(c_i | A) = \frac{p(c_i)p(A | c_i)}{\sum_i p(c_i)p(A | c_i)}$$



## Monty-Hall Dilemma

---



Suppose candidate chooses door 1 and quiz master opens door 2.  
Shall the candidate stay with the door chosen?



## Mathematics of Monty-Hall

---

Let  $D_t$  denote „prize is behind door  $t$ “.

Let  $O_t$  denote „quiz master opens door  $t$ “.

Priors:  $P(D_t) = 1/3$ ,  $t=1,2,3$ .

Conditional Probabilities:  $P(O_2|D_1)=1/2, P(O_2|D_2)=0, P(O_2|D_3)=1$ .

$P(O_2) = P(O_2|D_1) + P(O_2|D_2) + P(O_2|D_3) = (1/2 + 0 + 1)1/3 = 1/2$ .

**Bayes Theorem:** 
$$P(D_1 | O_2) = \frac{P(O_2 | D_1)P(D_1)}{P(O_2)} = \frac{1/6}{1/2} = 1/3$$



## x. Rational Time Preferences

---

- Time-Preference Reversal
- Hyperbolic Discounting
- Discounting and Risk Aversion



## Time-Preference Reversal

---

CEO has the following preferences

Over the next three years profit profiles:

$$(30,70,100) \succcurlyeq (50,50,100)$$

because she prefers increasing profits

$$(50,50,20) \succcurlyeq (30,70,20)$$

because she finds the drop from 70 to 20 to harsh.



## Time-Preference Reversal

---

CEO has the following preferences

$$(30,70,100) \succcurlyeq (50, 50, 100)$$

But lending 20 from first to second period

$$(-20,+20(1+r),0)$$

Makes  $(50,50,100)$  dominate  $(30,70,100)$

I.e. the CEO's choice contradicts Axiom 0!

With a perfect capital market one must discount with the interest rate!



## Hyperbolic Discounting

---

„From todays point of view one may find an action to be taken tomorrow profitable but when it comes to tomorrow this is no longer so.“

Examples: Quit smoking, start a diet, enroll in a gym, write a savings plan.

Hyperbolic discounting leads to a preference reversal that can be exploited in perfect capital markets, because capital markets discount exponentially not hyperbolicly.

Laibson, D. (1997) "Golden Eggs and Hyperbolic Discounting", Quarterly Journal of Economics, 112, 443-477.



## Hyperbolic Discounting

---

Utility Function:  $U_t(C_t, C_{t+1}, \dots, C_T) = C_t + \beta \sum_{\tau=t+1}^T C_\tau \delta^{\tau-t}$

where  $1 \geq \beta \geq 0$  and  $1 \geq \delta \geq 0$

Example: Taking an action at t that will result in  $C_t = -5$  and  $C_{t+1} = 6$ .

Point of view t-1: action is preferable if  $\beta(-5\delta + 6\delta^2) > 0 \Leftrightarrow \delta > \frac{5}{6}$

Point of view t: action is preferable if  $-5 + \beta 6\delta > 0 \Leftrightarrow \beta\delta > \frac{5}{6}$

Hence for  $\frac{1}{\beta} \frac{5}{6} > \delta > \frac{5}{6}$  we get a preference reversal.

Take for example  $\delta=1$  and  $\beta < 1$ .



## Discounting and Risk Aversion

---

Note that with standard Eu representation risk preference and time preference are closely linked:

Example: CRRA:

Q) How to define expected utility over time in which the risk preference can be distinct from the time preference?

A) Epstein and Zin (1989).

Epstein and Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns", *Econometrica* (57), pp.937-69.



## Discounting and Risk Aversion

---

Note that with standard Eu representation

$$U(c) = u(c_0) + \beta \sum_s p_s u(c_s)$$

risk preference and time preference are closely linked.

Example:

$$\text{CRRA: } u(c_s) = (c_s)^{1-\alpha}$$

$$\text{Time preference: } \frac{\partial_1 u(c_1)}{\partial_0 u(c_0)} = \beta \left( \frac{c_1}{c_0} \right)^\alpha \quad \text{Risk preference: } \frac{\partial_s u(c_s)}{\partial_0 u(c_0)} = \left( \frac{c_s}{c_0} \right)^\alpha$$

Epstein and Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns", *Econometrica* (57), pp.937-69.

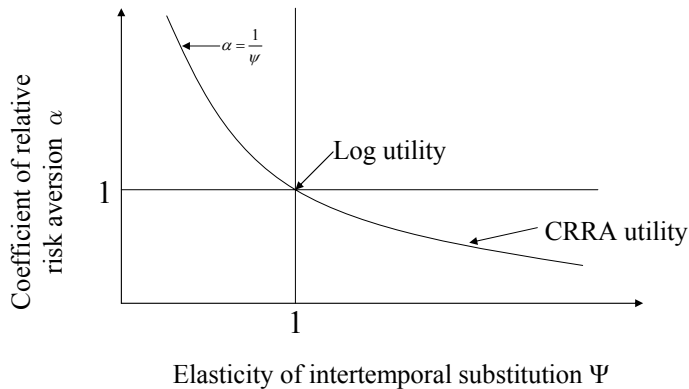


## Discounting and Risk Aversion

---

Epstein and Zin Representation of Eu:

$$U_t(C) = \left[ (1-\delta)C_t^{\frac{(1-\alpha)}{\theta}} + \delta(E_t U_{t+1}^{1-\alpha})^{\frac{1}{\theta}} \right]^{\frac{\theta}{(1-\alpha)}} \quad \text{where } \theta \equiv \frac{(1-\alpha)}{(1-\frac{1}{\psi})}$$



## References on Rational Choice

---

- Huang Litzenger (1988) Chapter 1.
- Howard (1988): Decision Analysis, Management Science (34), pp. 679-697.
- Keeney (1982): Decision Analysis, Operations Research (30), pp. 803-838.

