University of Zurich
Institute for Empirical Research in Economics

Advanced Portfolio Theory

NHH-Bergen

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Universität Zürich

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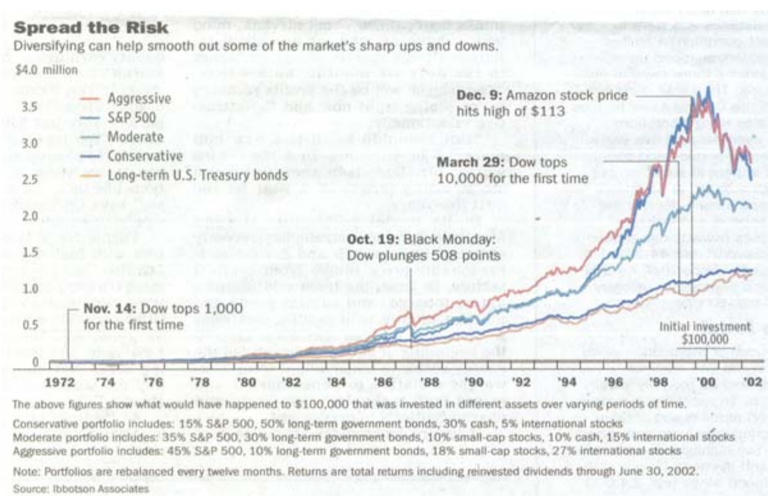
7. Chosing a Portfolio on a Mean-Reverting Process: Timing

- Empirical Evidence for Mean Reversion
- An Time Series Model for Mean-Reversion
- Asset Pricing Models explaining Mean-Reversion
- Optimal Portfolio Choice (Campbell Viciera)
- Life Cycle Planning

a) Equity Premium Puzzle (1)



a) Equity Premium Puzzle (2)

Table 2 Returns for Selected Countries, 1947 – 1998				
Country	Time period	% real return on a market index	% real return on a relatively riskless security	% risk premium
		Mean	Mean	Mean
U.K	1947-1999	5.7	1.1	4.6
Japan	1970-1999	4.7	1.4	3.3
Germany	1978-1997	9.8	3.2	6.6
France	1973-1998	9.0	2.7	6.3

Source: U.K from Siegel (1998), the rest are from Campbell (2001)

Table 3 Terminal Value of $1 Invested in:				
Investment period	Stocks		T-bills	
	Real	Nominal	Real	Nominal
1802-1997	$558,945	$7,470,000	$276	$3,679
1926-2000	$266.47	$2,586.52	$1.71	$16.56

Source: Ibbotson (2001) and Siegel (1998)

a) Why is this fact called Equity Premium **Puzzle** ?

Which X would you make indifferent to the following lottery?

Dobling your income with 50% probability
Loosing X% of your income with 50% probability

Typical answer: X = 23%

Expected utility maximizing investor who chooses the risk free asset instead of stocks chooses X = 4 %.

a) Equity Premium **Puzzle** (3)

Contradict the Random Walk Hypothesis:

The Variance of a random walk pay off is proportional to time.

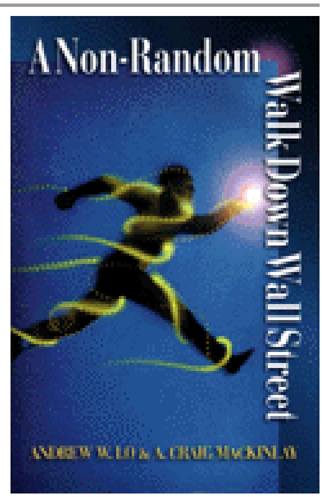
Statistical justification of
Equity Premium Puzzle: Mean Reversion!

a) New Econometric Evidence

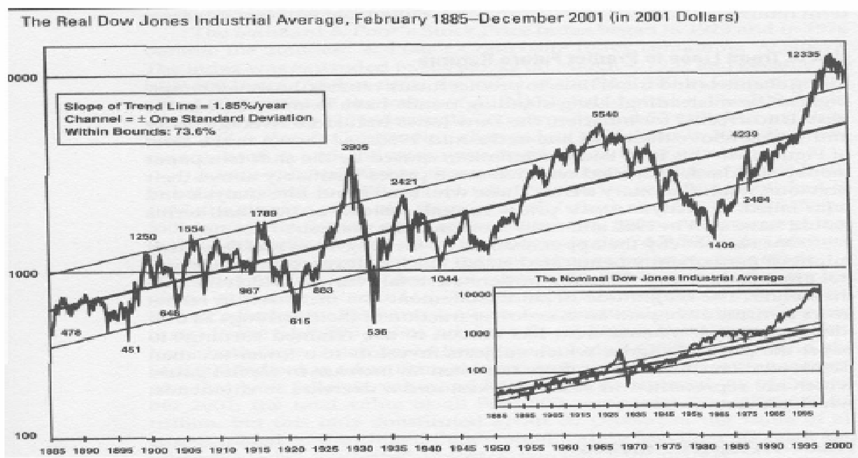
Financial Econometrics:

Lo & MacKinley (1999):

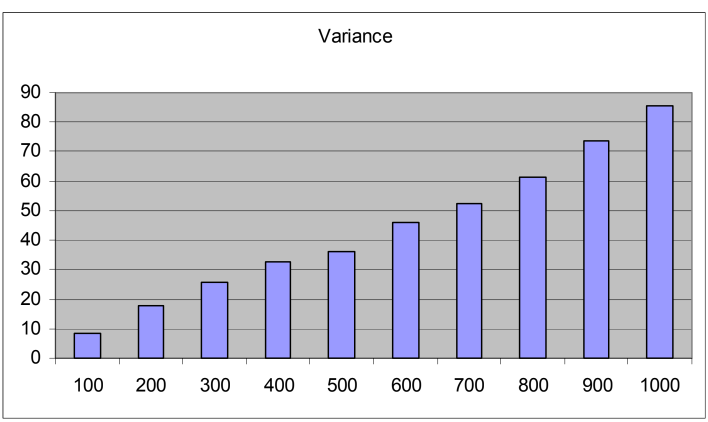
- Momentum & Reversal
- Over- & Underreaction



a) Mean Reversion on DJIA

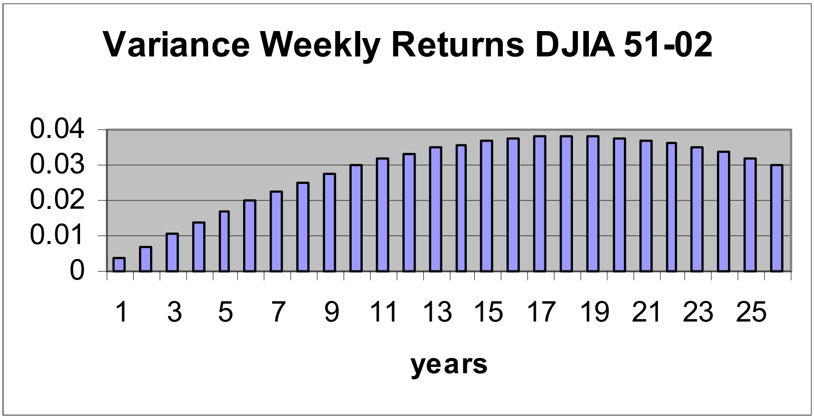


a) Evidence for Mean Reversion



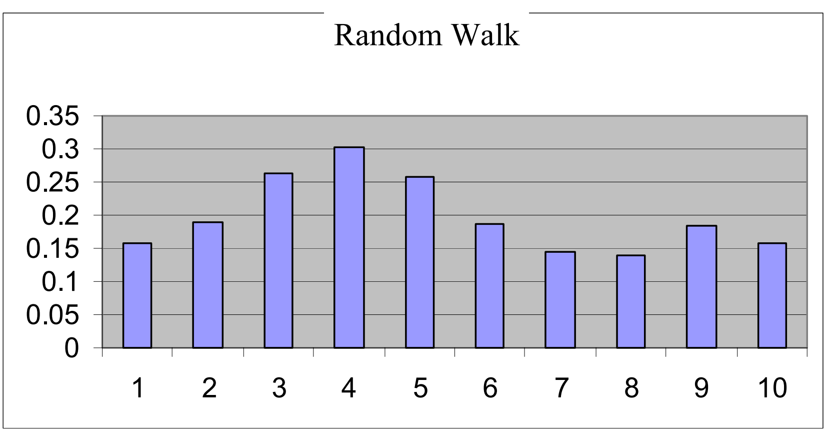
The variance of a random walk increases linearly with time

a) Mean Reversion on DJIA (1)



… are increasing less than linearly!

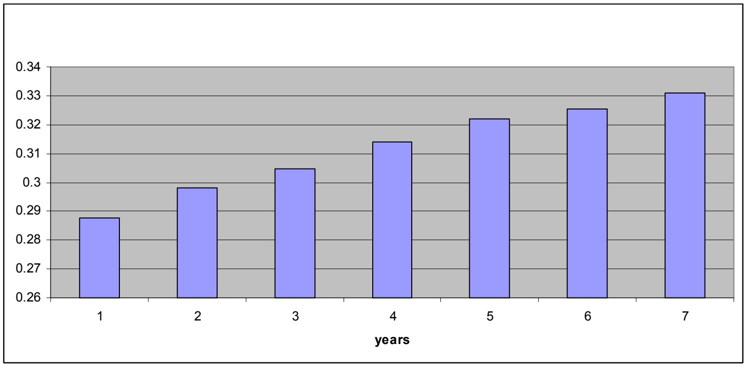
a) Evidence on Mean Reversion



Ratio of expected return to variance of a random walk has no clear pattern

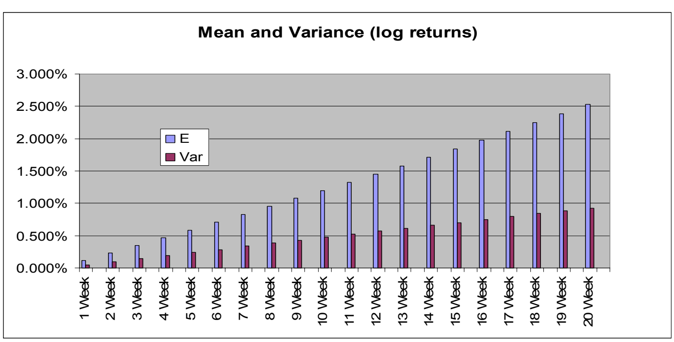
a) Mean Reversion on DJIA Weekly 1951-2002

Ratio Expected Return to Variance



.. is increasing over time !

Evidence for Mean Reversion



Ratios of Means to Variances **are increasing over time !**

b) Mean-Reverting Processes

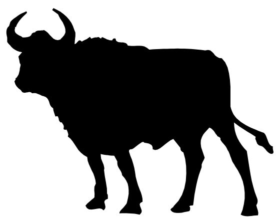
- Definition as AR(1)
- See Campbell Viceira.

Shiller (2000): We must look elsewhere

"In sum, stock prices clearly have a life of their own; they are not simply responding to earnings or dividends. Nor does it appear that they are determined only by information about future earnings or dividends. In seeking explanations of stock price movements, we must look elsewhere."

Shiller (2000): "Irrational Exuberance",

b) A Model: Rational Investor Sentiment



Anke Gerber Thorsten Hens and Bodo Vogt
(IEW-University of Zurich)

Barberis, Shleifer, Vishny (1998)

A Model of Investor Sentiment:

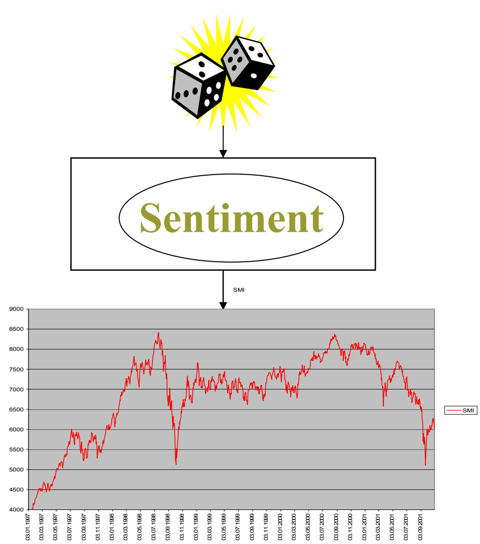
> Earnings follow a random walk but investors believe the market switches between two regimes: a ``momentum'' and a ``mean-reversion'' state. If investors do Bayesian updating every period then they switch between two moodes: overreaction and underreaction.

"While we do modify the investor's preferences to reflect experimental evidence about the sources of utility, the investor remains fully rational and dynamically consistent throughout[5]."

[5] see Shleifer, 1999 for a recent treatment of irrationality in financial markets:

Barberis, Huang, Santos (1999),
Prospect Theory and Asset Prices

Investor Sentiment Hypothesis



A Trader‘s point of view

„Ninety percent of what we do is based on perception.
It doesn‘t matter if that perception is right or wrong or real. It only matters that other people in the market believe it.

I may know it‘s crazy, I may think it‘s wrong. But I lose my shirt by ignoring it.

This business turns on decisions made in seconds. If you wait a minute to reflect on things, you‘re lost. I can‘t afford to be five steps ahead of everybody else in the market. That‘s suicide.“

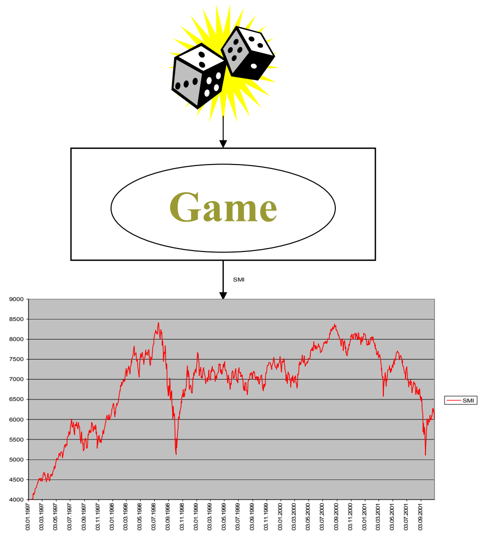
“Making Book on the Buck”
Wall Street Journal, Sept. 23, 1988, p. 17

Gerber, Hens and Vogt (2002)

Coordination Game:

View the stock market as a repeated coordination game with imperfect monitoring, where exogenous noise as well as the investor sentiment determine the stock price.

Coordination Game Hypothesis



The Experiment

Stage Game:

5 Participants bet 1 point each on Up or Down

Dice randomly distributes 6 points on Up or Down

Price movement: Up *if* # Up > # Down

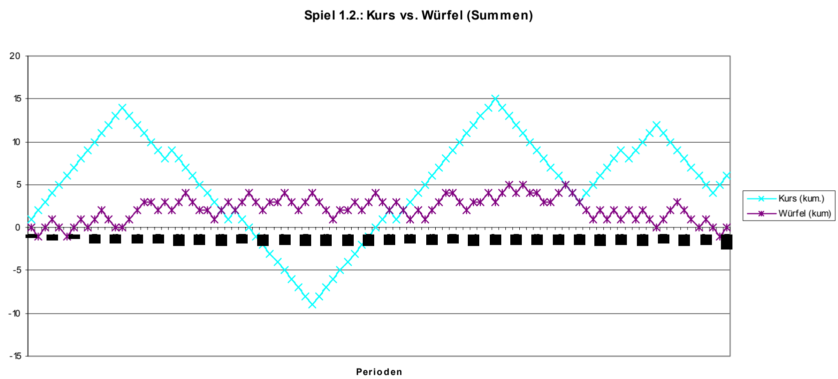
Down *if* # Down > # Up

Pay off : If prediction correct then 20 ECU otherwise noting

The stage game is repeated in two rounds with 100 periods each

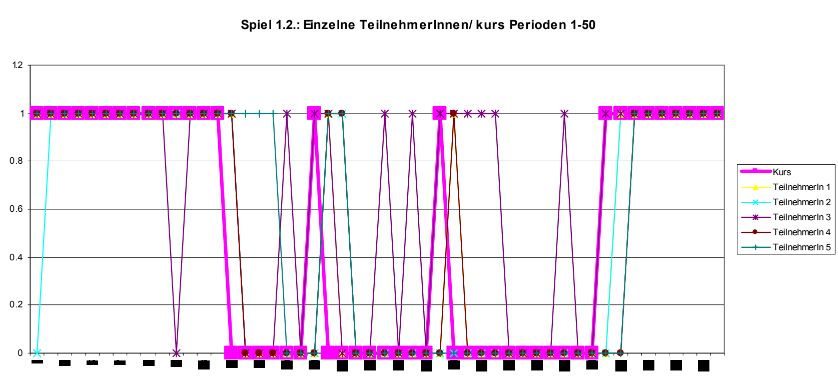
Experimental Analysis

Result of Laboratory Experiment:



Experimental Analysis

Result Laborexperiment:



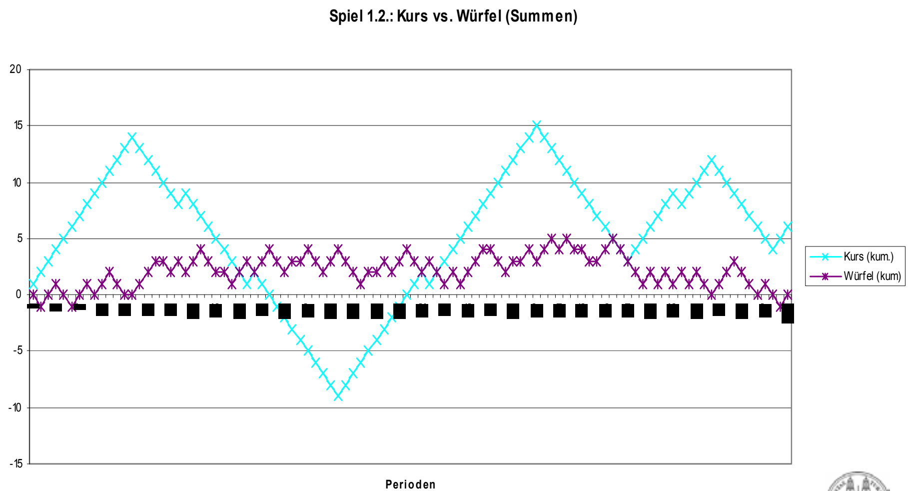
More Results in File

Up&DownGrafiken.pdf

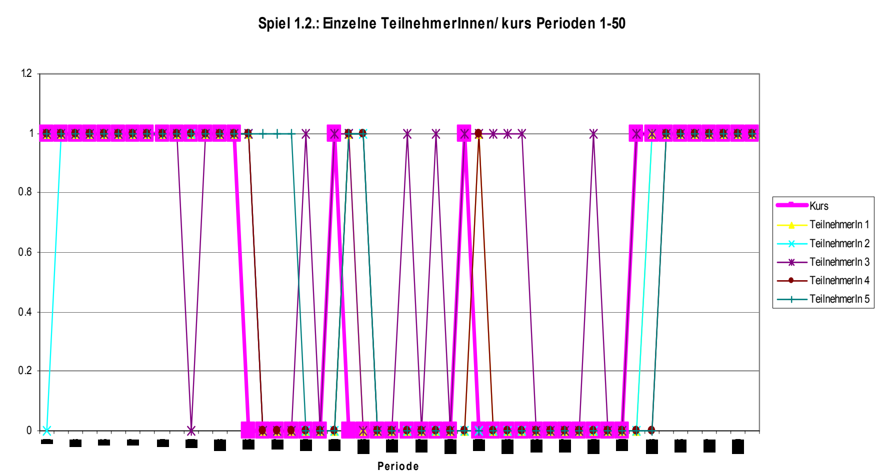
Two possible explanations for switching

- Focal Point Analysis
- Probability Matching

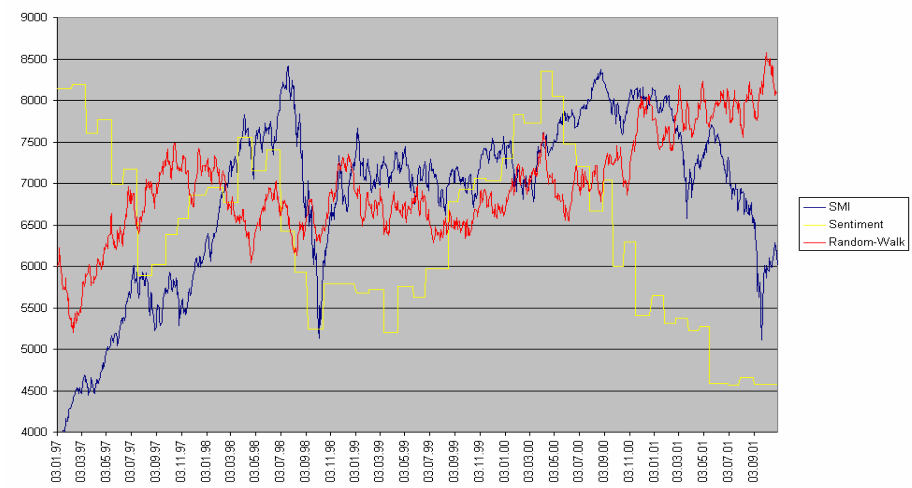
Experimental Evidence: Momentum and Reversal and Excess Volatility



Experimental Evidence

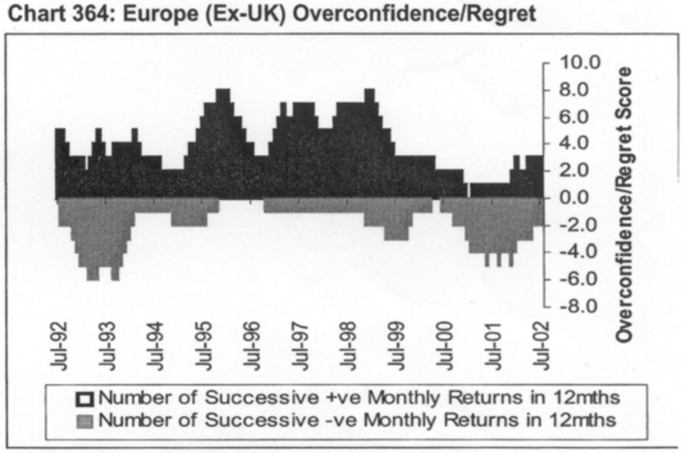


Sentiment-Index of CS



Merrill Lynch Overconfidence Regret Potential

Chart 364: Europe (Ex-UK) Overconfidence/Regret



Source: Datastream, Merrill Lynch

c) Optimal Portfolio Choice with Mean Reversion

Claim: A rational investor with reasonable risk aversion should hold more stocks when returns are mean reverting!

Distinguish:

- Myopic versus long term planning
- Buy and hold versus timing

e.g. *myopic buy and hold*: you plan one period ahead and don`t try to time

tactical asset allocation: you plan one period ahead but also try timing

strategic asset allocation: you plan over multiple periods and do timing

c) Optimal Portfolio Choice with Mean Reversion

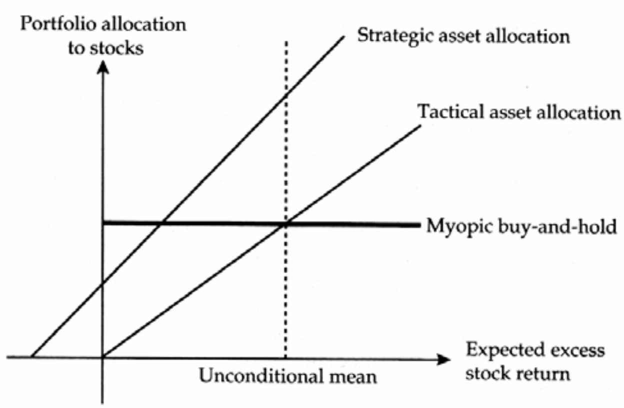


Figure 4.1. Alternative portfolio rules.

Campbell and Viceira (2002):"Strategic Asset Allocation", page 99.

Asset Allocation and Mean Reversion

Siegel (1994): "Stocks for the Long Run", McGraw-Hill, p.33:

"Stocks have what economists call mean-reverting returns, meaning that over long periods of time, high returns seem to be followed by periods of low returns and vice versa. On the other hand, over time, real returns on fixed income assets become relatively less certain. For horizons of 20 years or more, bonds are riskier than stocks."

From this Siegel follows what also Kostolyani recommends: "Buy stocks and take a good long (20 years) sleep."

Problem: Mean reversion means predictability! Hence you can do even better by timing the market.

Myopic Loss Aversion

- Thaler and Johnson
- Derive it from PT-non convexity

Mean Reversion and Asset Allocation

Proof of the claim:

Campbell and Viceira (2002):

AR(1) process for log-normal returns and Epstein-Zin Utility

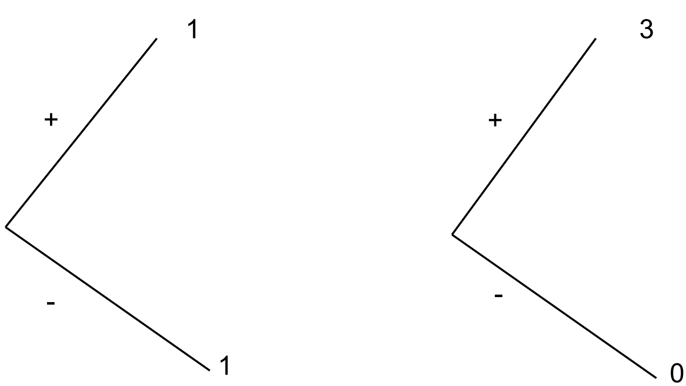
Samuelson (1991):

2 states Markov process and CRRA – Expected Utility

Samuelson (1991): The Canonical Case

2 states 2 assets

Returns:



Safe cash

equity

Samuelson (1991): Markov Process

t-1 \ t	+	-
+	b	1-b
-	c	1-c

Mean-reversion

t-1 \ t	+	-
+	1/3	2/3
-	2/3	1/3

Random walk

t-1 \ t	+	-
+	1/2	1/2
-	1/2	1/2

Momentum

t-1 \ t	+	-
+	2/3	1/3
-	1/3	2/3

Samuelson (1991): Utility Functions

CRRA-class:

$$U(W) = \frac{W^{\alpha}}{\alpha}, 0 \neq \alpha < 1$$

Most realistic:

U(W) = -1/w

Bernoulli:

U(W) = ln(W)

Cramer:

$$U(W) = \sqrt{W}$$

Samuelson (1991): Bernoulli Utility (I)

- Myopic optimization
- Tactical asset allocation
- Strategic asset allocation

Samuelson (1991): Bernoulli Utility (II)

- Myopic optimization

$$\underset{\lambda}{Max}\; p\ln(1+2\lambda)+(1-p)\ln(1-\lambda)$$

$$Solution: \lambda^* = \frac{3p-1}{2}$$

p=1/3 → *After good realisation:* $\lambda_-^* = 0$

p=1/2 → *RandomWalk:* $\lambda^* = 1/4$

p= 2/3 → *After bad realisation:* $\lambda_-^* = 1/2$

Samuelson (1991): Bernoulli Utility (III)

First Result:

For a myopic Bernoulli utility maximizer

the long run average of the timing asset allocation

coincides with the asset allocation on a random walk

that has the same long run probabilites as the markov process.

Proof in the example above:

$$1/2\ \lambda^*_- + 1/2\ \lambda^*_+ = \lambda^*$$

$$1/2\ 0 + 1/2\ 1/2 = 1/4$$

Samuelson (1991): Bernoulli Utility (IV)

- Long Term Planning: Strategic Asset Allocation

  Let $\omega_t \in \{+,-\}$ be the realisation in period t.

  And let $\omega^t = (\omega_0,...,\omega_t)$ be the history up to period t.

  Then the evolution of wealth can be written as:

  $w^{t+1}(\omega^{t+1}) = [1-\lambda^t(\omega^t) + R(\omega_{t+1})\lambda^t(\omega^t)]\ w^t(\omega^t)$

  Where $\lambda^t(\omega^t)$ is the decision taken at t and $R(\omega_{t+1})$ the return at t+1.

  Evaluating end of planning horizon wealth $w^T(\omega^T)$ by ln

  we see that the T-periods planning problem decomposes in

  T separate one period problems:

  $\max_{\lambda^t(\omega^t)} \ prob(+|\omega^t)\ln(1-2\lambda^t(\omega^t)) + prob(-|\omega^t)\ln(1-\lambda^t(\omega^t))$

  Hence we get the same optimal solution λ^*_-, λ^*_+ as before.

Samuelson (1991): Bernoulli Utility (III)

<u>Second Result:</u>

A strategic long-term Bernoulli utility maximizer

chooses the same asset allocation

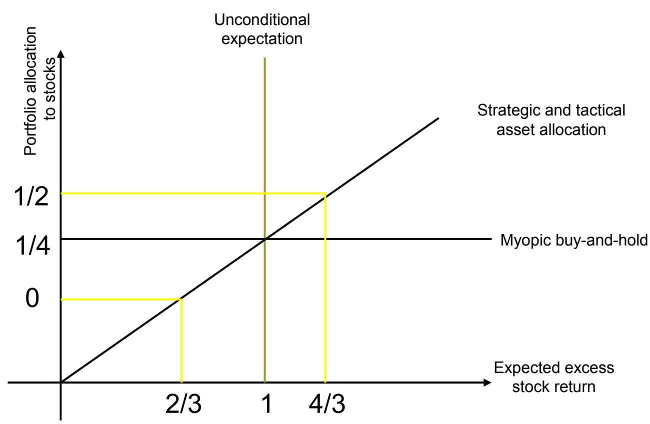
as the myopic Bernoulli utility maximizer.

Proof :

See previous slide.

Samuelson (1991): Bernoulli Utility (IV)

- Summary



Alternative portfolio rules for Bernoulli

Samuelson (1991): More Realistic Utility (I)

- Myopic optimization
- Tactical asset allocation
- Strategic asset allocation

Samuelson (1991): More Realistic Utility (II)

- Myopic optimization

$$\underset{\lambda}{Max}\ -p(1+2\lambda)^{-1}-(1-p)(1-\lambda)^{-1}$$

$$Solution: \lambda^* = \begin{cases} \frac{1}{4} & p = 2/3 \\ \frac{4+\sqrt{72p(1-p)}}{4(3p-2)} & 0 < p < 2/3 \\ \frac{4-\sqrt{72p(1-p)}}{4(3p-2)} & 2/3 < p < 1 \end{cases}$$

p=1/3 → *After good realisation:* $\lambda^*_- = 0$

p=1/2 → *Random Walk:* $\lambda^* = 0.1213...$

p= 2/3 → *After bad realisation:* $\lambda^*_- = 1/4$

Samuelson (1991): More Realistic Utility (III)

Third Result:

For a more realistic utility maximizer

the long run average of the timing asset allocation

is **greater** than the asset allocation on a random walk

that has the same long run probabilites as the markov process.

Proof in the example above:

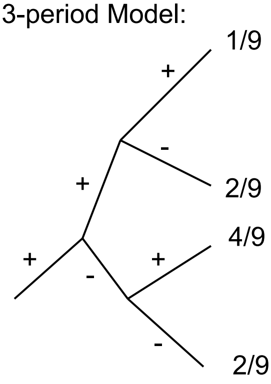
$$1/2\ \lambda_-^* + 1/2\ \lambda_+^* > \lambda^*$$

$$1/2\ 0 + 1/2\ 1/4 > 0.1213...$$

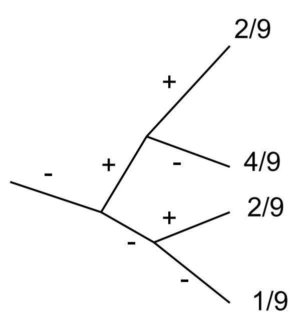
Samuelson (1991): More Realistic Utility (IV)

Long Term Planning: Strategic asset allocation:

3-period Model:



Case 1



Case 2

Samuelson (1991): More Realistic Utility (IV)

Long Term Planning: Strategic asset allocation

By backward induction:

Case 1: Let $\lambda_-^2 (\lambda_+^2)$ be the optimal one period choice in the second period after a -(+) return.

Determine the optimal first period choice λ_+^1 after a + return just before:

$$\max_{\lambda} \; -1/9\,(1+2\lambda)^{-1}(1+2\lambda_+^2)^{-1} - 4/9\,(1-\lambda)^{-1}(1+2\lambda_-^2)^{-1} - 2/9\,(1+2\lambda)^{-1}(1-\lambda_+^2)^{-1} - 2/9\,(1-\lambda)^{-1}(1-\lambda_+^2)^{-1}$$

$$= \frac{25}{27} \max_{\lambda} \; -9/25\,(1+2\lambda)^{-1} - 16/25\,(1-\lambda)^{-1}$$

Hence taking into account the second period optimization

the odds for a good outcome have changed from 1 to 2 in the myopic case to 16 to 25.

Consequently, the investor invest more in the risky asset: $\lambda_+^1 = 0.0198193 > 0$.

Samuelson (1991): More Realistic Utility (IV)

Long Term Planning: Strategic asset allocation

By backward induction:

Case 2: Let $\lambda_-^2 (\lambda_+^2)$ be the optimal one period choice in the second period after a - (+) return.

Determine the optimal first period choice λ_-^1 after a - return just before:

$$\max_{\lambda} \; -2/9\,(1+2\lambda)^{-1}(1+2\lambda_+^2)^{-1} - 4/9\,(1+2\lambda)^{-1}(1-\lambda_+^2)^{-1} - 2/9\,(1-\lambda)^{-1}(1+2\lambda_-^2)^{-1}(1-\lambda_+^1)^{-1} - 1/9\,(1-\lambda)^{-1}(1-\lambda_+^2)^{-1}$$

$$= \frac{26}{27} \max_{\lambda} \; -18/26\,(1+2\lambda)^{-1} - 8/26\,(1-\lambda)^{-1}$$

Hence taking into account the second period optimization

the odds for a good outcome have changed from 1 to 2 in the myopic case to 18 to 26.

Consequently, the investor invest more in the risky asset: $\lambda_+^1 = 0.272078... > 0.25$

Samuelson (1991): More Realistic Utility (V)

Fourth Result:

A strategic long-term more realistic utility maximizer

chooses an asset allocation that **has more risky assets** than

the myopic more realistic utility maximizer.

Proof :

See previous slide.

Samuelson (1991): Cramer`s Utililty

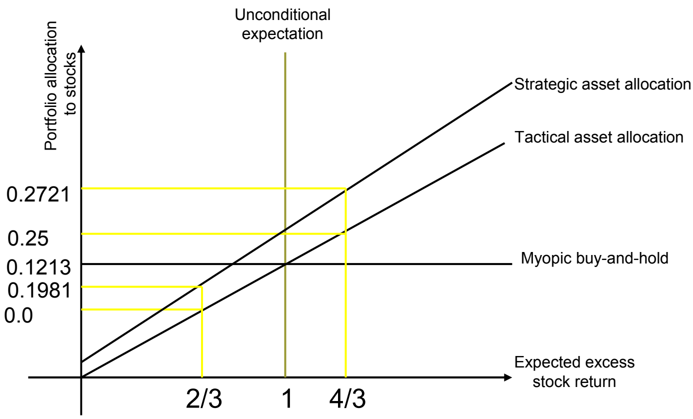
The more risk averse utility U(W) = -1/W has a **smaller** allocation of risky assets than the Bernoulli case and the longer the time horizon the **greater** the proportion of risky assets.

The less risk averse utility $U(W) = \sqrt{W}$ has a **greater** allocation of risky assets than the Bernoulli case and the longer the time horizon the **smaller** the proportion of risky assets.

For the more risk averse utility U(W) = -1/W we observe **time diversification**, i.e. the allocation of risky assets increase with the investment horizon.

Samuelson (1991): More Realistic Utility (VI)

- Summary



Alternative portfolio rules for more realistic utility

Conclusion: Asset Allocation with Mean Reversion

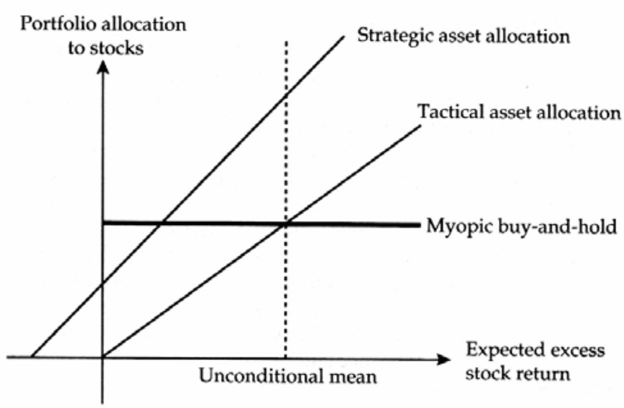


Figure 4.1. Alternative portfolio rules.

Campbell and Viceira (2002):"Strategic Asset Allocation", page 99.

Life Cycle Planning (I)

Issues involved:

- Life cycle income
- Retirement System
- Borrowing constraint on human wealth
- Background risk: housing and private equity
- Risk and Time Preferences

Campbell and Viciera (2002) Chapter 7

Life Cycle Planning (II)

- A Life-Cycle Model of Portfolio Choice

Time Parameters and Preferences for individual i:

$$\frac{C_{i,t}^{1-\alpha^i}}{1-\alpha^i} + E_t \sum_{\tau=1}^{T-\tau} (\delta^i)^\tau \left(\prod_{j=0}^{\tau-1} p^i{}_{t+\tau} \right) \frac{C_{i,t+\tau}^{1-\alpha^i}}{1-\alpha^i}$$

where T maximum life time

α^i risk preference

δ^i time preference

$p^i{}_{t+\tau}$ survival probability

Campbell and Viciera (2002) Chapter 7

Life Cycle Planning (III)

- A Life-Cycle Model of Portfolio Choice

  Labor income process for individual i:

  $l_{i,t} = f(t, Z_{i,t}) + v_{i,t} + \varepsilon_{i,t}$

  where all variables are in logs

  $f(t, Z_{i,t})$ deterministic function of age and other individual characteristics $Z_{i,t}$

  $\varepsilon_{i,t} \sim \mathrm{N}(0, \sigma_\varepsilon^2)$ idiosyncratic temporary shock

  $v_{i,t}$ is given by $v_{i,t} = v_{i,t-1} + u_{k,t}$

  where $u_{k,t} \sim \mathrm{N}(0, \sigma_u^2)$ is uncorrelated with $\varepsilon_{i,t}$

  and $u_{k,t} = \xi_t + \omega_{i,t}$ is aggregate and idiosyncratic

Campbell and Viciera (2002) Chapter 7

Life Cycle Planning (IV)

- A Life-Cycle Model of Portfolio Choice

  Financial assets: Riskless and risky

  $R_{t+1} - R_f = \mu + \eta_{t+1}$

  where μ is a deterministc drift

  $\eta_{t+1} \sim \mathrm{N}(0, \sigma_\eta^2)$ is an i.i.d. disturbance.

  $f(t, Z_{i,t})$ deterministic function of age and other individual characteristics $Z_{i,t}$

  $\varepsilon_{i,t} \sim \mathrm{N}(0, \sigma_\varepsilon^2)$ idiosyncratic temporary shock

  $v_{i,t}$ is given by $v_{i,t} = v_{i,t-1} + u_{k,t}$

  where $u_{k,t} \sim \mathrm{N}(0, \sigma_u^2)$ is uncorrelated with $\varepsilon_{i,t}$

  and $u_{k,t} = \xi_t + \omega_{i,t}$ is aggregate and idiosyncratic

Campbell and Viciera (2002) Chapter 7

Life Cycle Planning (V)

- A Life-Cycle Model of Portfolio Choice

Retirement and liquid wealth:

$L^d_{i,t} = (1-\theta)L_{i,t}$ for $t \leq S$

where $L^d_{i,t}$ is disposable income

θ is fraction going to retirement wealth $W^R_{i,t}$.

At S retirement wealth is rolled into a riskless annuity.

Assume: Retirement wealth is hold in a riskless asset.

Borrowing constraint on rsikless asset.

Short sales constraint on risky asset.

Fixed cost to enter the market for risky asset.

Life Cycle Planning (VI)

- A Life-Cycle Model of Portfolio Choice

The individual`s optimization problem:

For $t \leq S$ the evolution of disposable and of retirement wealth is:

$$W^d_{i,t+1} = [1+\lambda_{i,t}R_{t+1} + (1-\lambda_{i,t})R_f][W^d_{i,t} + (1-\theta)L_{i,t} - C_{i,t}]$$

$$W^R_{i,t+1} = (1+R_f)[W^R_{i,t} + \theta L_{i,t}]$$

For $t > S$ the retirement wealth is annuitized to $A\left(W^R_{i,t}\right)$.

Life Cycle Planning (VII)

- Characteristics of representative US-households

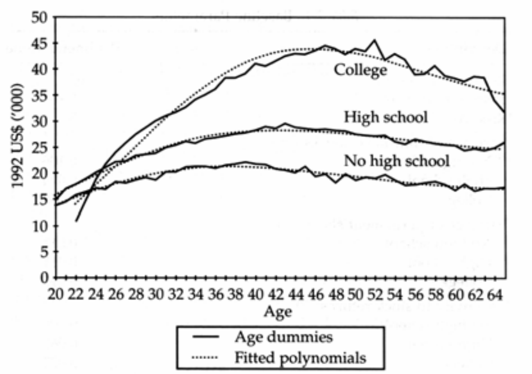


Figure 7.1. Labor income profiles.

Campbell and Viciera (2002) Chapter 7

Life Cycle Planning (VIII)

Characteristics

of representative

US-households

Table 7.1. Baseline Parameters

Description	Parameter value
Retirement age (K)	65
Discount factor (δ)	0.96
Risk aversion (γ)	5
Variance of transitory shocks (σ_ε^2)	
No high school	0.106
High school	0.074
College	0.058
Variance of permanent shocks (σ_u^2)	
No high school	0.011
High school	0.011
College	0.017
Sensitivity to stock returns (β)	
No high school	0.096
High school	0.063
College	0.073
Correlation with stock returns ($\rho_{\xi\eta}$)	
No high school	0.328
High school	0.371
College	0.516
Riskless rate ($\bar{R}_F - 1$)	0.02
Mean excess return on stocks (μ)	0.04
Standard deviation of stock return (σ_η)	0.157
Fixed cost (F)	0 or 10,000
Social security tax rate (θ)	0.10

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Life Cycle Planning (VIII)

- Benchmark results

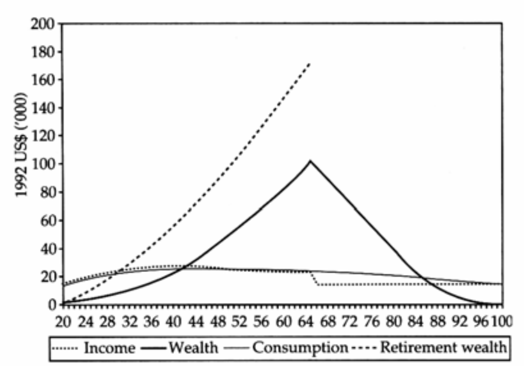

Figure 7.2. Consumption, income, and wealth

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Life Cycle Planning (VIII)

- Benchmark results

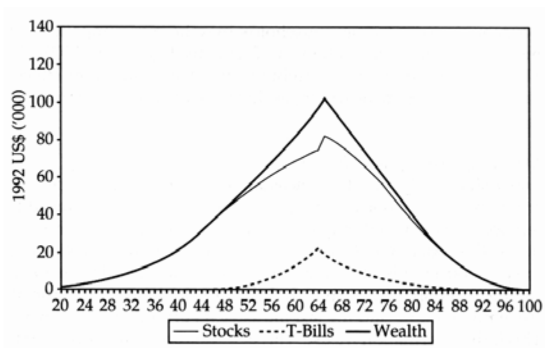

Figure 7.3. Liquid wealth, stocks, and Treasury bills.

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Heterogeneity

Table 7.4. Life-Cycle Profiles

Age	Baseline	$\gamma = 10$	$\delta = 0.8$	Self-employed
Consumption				
20–35	20.22	20.13	20.53	25.09
36–50	25.48	25.12	26.50	38.39
51–65	24.61	24.23	23.94	35.23
66–80	22.43	22.65	15.95	32.67
81–100	16.98	19.04	14.27	27.26
Wealth				
20–35	5.94	8.20	3.39	12.84
36–50	29.34	39.28	7.25	65.75
51–65	75.77	100.16	10.23	173.70
66–80	77.28	105.50	5.71	159.76
81–100	13.60	30.85	0.11	46.75
Liquid portfolio share in stocks				
20–35	1.00	0.97	0.99	0.57
36–50	0.99	0.95	1.00	0.91
51–65	0.88	0.61	1.00	0.57
66–80	0.90	0.57	1.00	0.54
81–100	0.92	0.68	1.00	0.61

Individual Characteristics Matter

Table 6.1: Wealthy Individual

Henry Highgate is a sixty-year-old business owner who has sold his business to a public company in the last year. He holds a substantial amount of the acquiring company's stock and is an aggressive investor with his remaining wealth. Henry's wife, Mary, is fifty-eight, and his son, James, is in high school and is sixteen. Henry relies on the advice of his attorney, accountant, and several brokers, private bankers, and money managers. Henry is a sophisticated investor who is in the highest tax bracket (49%). Henry wants 6,000,000 in cash flow annually and has no other sources of income except for his assets.

Assets	Tax Basis	Value	Cash Flow
Cash & Marketable Securities	29,523,641	41,047,448	1,231,423
Closed-End Funds	38,642,851	60,088,774	660,977
Restricted Stock	195,453,200	286,722,100	661,000
Illiquid Assets	1,195,275	2,751,915	-
Insurance	1,006,000	1,142,117	-
Limited Partnerships	5,364,780	12,806,664	650,000
Commercial Real Estate	165,300,000	205,400,000	12,000,000
Personal Real Estate	18,500,000	22,400,000	-
Total Assets	454,985,747	632,359,018	15,203,400
Liabilities & Equity	Value	Cash Flow	
Credit Cards & ST Loans	87,543	(87,543)	
Margin Loans	10,324,312	(2,100,000)	
Commercial R/E Loans	82,160,000	(7,394,400)	
Personal R/E Loans	15,680,000	(1,411,200)	
Unfunded Equity Commitments	88,878,692	-	
Total Liabilities	197,130,547		
Net Equity	435,228,471		
Total Liabilities and Equity	632,359,018	(10,993,143)	
Current annual income			15,203,400
Current annual expenses & liabilities			(10,993,143)
Net cash flow			4,210,257
Cash flow required			(6,000,000)
Cash Flow Shortfall			(1,789,743)

Individual Characteristics Matter

Table 6.2: Family Office

John James manages a family office with several family members as his full time job. The family's wealth is held in various liquid and illiquid assets. James is particularly interested in the net tax effects of investment choices he can make. He is not concerned about short term shifts in individual parts of the family portfolio as he wants to make all decisions with a long term perspective

Several individuals	Tax bracket	Income Needs	
Individual John James	49%	650,000	
Individual Susan James (sister)	49%	800,000	
Individual Frank James (brother)	15%	1,000,000	
Individual Michael James (son)	37.6%	250,000	
Total income required		2,700,000	
Assets	Value	Tax Basis	Cash Flow
Hedge Funds	4,500,000	3,400,000	0
US Equity Funds	6,700,000	3,600,000	107,200
Foreign Funds	3,800,000	2,700,000	68,400
Taxable Bonds	3,500,000	3,100,000	227,500
Tax Exempt Bonds	6,000,000	4,700,000	324,000
Private Equity	6,400,000	850,000	0
Family Business	35,000,000	1,200,000	850,000
Family Trust (untouchable)	40,000,000	1,100,000	220,000
Real Estate	6,500,000	1,400,000	150,000
Totals	112,400,000	22,050,000	1,947,100
Liabilities			
Family Business Debt	15,000,000	15,000,000	(1,312,500)