

6. Evolutionary Portfolio Theory – Professor Thorsten Hens

Thank you for inviting me. To figure out the first difference in the welcome and opening remarks, the headline was “moving from the normative to the positive”. My talk is to deal with the normative, because it is important to give advice and to think about the best trading rules and not simply to describe how people behave like in behavioural finance or to generate models which give us the same type of anomalies as we find in data. We want to get some advice from the researchers, which is why you pay us, why you go to university. That is the good news of my presentation and the bad news is that I shall not give you very precise advice in the sense that I shall tell you buy this talk or buy the others’ talk. However, I want to give you a new way of framing portfolio decisions where these ideas are borrowed from evolutionary biology, so after having econophysics we now have econobiology. What we do is the Darwinian story of the stock market, which is the *Survival of the Fittest at Wall Street*.

At this stage, I should mention that I have two co-authors: Igor Evstigneev from Manchester University and Klaus Schenk-Hoppe who now moves to Copenhagen University. We will come back to those names because my presentation has a very traditional form. I make simplifying assumptions and I do not show you simulations but I prove two theorems, because it is important to understand what is the reason for what.

Let us look at the analogy between biology and finance. This is the analogy which we draw: survival of the fittest at Wall Street. The first thing that is important in biology is that the different agents are described by the strategy that they follow. It is not important whether this rabbit has a certain appearance, what is important is whether this animal is playing the strategy of the rabbit or that of a fox, the strategy of a hawk or that of a dove. This is the first and most important shift of your thinking, because you do not want to look at the performance of individual investors but at the performance or strategies.

In finance, the strategy, at least in this example, is the portfolio rule. Therefore, we want to see what is the performance of the mean variance rule as compared to the condition of the condition value at risk rule, the expected shortfall rule, as compared to the behavioural finance rule based on prospect theory or whatever. The individual investors may well switch those rules, so that they may sometimes use this rule and sometimes another rule. First, we want to look at the market in terms of rules.

Why is this important? What drives the prices is not who is buying or who is selling but according to which rule certain wealth is managed. The second ingredient is resources, so whereas in biology certain animals are fighting for food or other types of resources, and in finance you and I are fighting for one sort of food which is money. Therefore, the resource for which you will compete is the market capital.

Then there are two forces in this evolutionary type of reasoning. One force is something which makes a certain ecology or certain population of strategy simpler, that is the selection force. The selection goes by gains and losses. If somebody makes a huge gain, you can be sure that there must be somebody who has made a huge loss. The selection process makes the population simpler, because some strategies will die out but then there is this countervailing force of mutation. Whenever some pool of strategies may become too simple, too narrow, for a certain reason there are other strategies popping up and invading this pool.

■ Evolutionary Finance: An Illustrative Example – Asset Allocation on DJIA

We have this idea, the strategy, the resources, the selection and the mutation as the two countervailing forces. This may be a little abstract, so in order to keep you awake after our nice lunch, you should participate in this evolutionary finance

issue. I have a nice example and everyone of you will now pick one of the most prominent portfolio rules and we define an ecology in this way. We define how many people have chosen CAPM rule, how many the mean variance rule and so on, then we do a contest in the sense that we let you play against each other on a stylized market.

Let us start with this example and afterwards I shall give the model, I will show the theorem though I shall not prove the theorem. We start from a known dividend process. Known is some exogenous dividend process. We have collected data on the Dow Jones Industrial Average and collected what was the total amount of dividends paid out by the firms in the DJIA. So it is not the dividend per share but the total amount of dividends they have been paying out to all the shareholders from 1981 to 2000. Now we assume that this dividend process is stationary, which is a terrible assumption for which I am sure you will kill me. However, I shall talk afterwards about what happens if it is non-stationary.

According to this type of random process, dividends are paid. However, you do not know the returns, because they are the dividends plus or minus the capital gains. The returns depend on the prices and these are determined by your strategies. So you are playing a game in a certain sense, this is evolutionary game theory if you like, against the others in this population who are the men and women in this room.

The task is to choose what we call a portfolio rule and to make things simple, before we move to the advanced things, we choose a simple portfolio rule. What is a simple portfolio rule? This is a constant division of your wealth according to the various assets which you can buy. Let me go through this in order.

■ 1) DJIA – Dividend Process

This is the dividend process. Here you see something like 20 or 29 such dividend policies which have been observed. Here you see the lock-up dividends, there is a growth rate in the dividend payments. Some firms follow a very smooth dividend policy like this firm, and some firms love to go up and down with their investors like this firm. Now you have to pick stocks once you know this dividend process.

■ 2) Returns = Dividends + Capital Gains

The story goes on and I shall show you now how the prices are determined. The prices are determined by what we have learned in economics for hundreds of years by supply and demand. We have considered the dividends as the total dividend payments of those firms so, in a sense, we have normalized supply to be equal to 1 by the way we have considered the dividends. Then, if you denote by $\lambda_{i,k,t}$ the percentage of wealth which strategy i invest in asset k in period t , then the price is simply this formula. There is no assumption, it is just by definition of the λ , the price of some asset k in period t is simply the average strategy which is played in this pool of strategies. So the price of asset k is the wealth average of strategies portfolio share of that asset.

This means, for example, that if we have a monomorphic population where everybody plays the same strategy as in the Markowitz model, where everybody plays mean variance, then the price has to be in accordance with only one rule according to the mean variance rule. However, if you have a heterogeneous population, the impact of every single strategy is proportional simply to the wealth according to which the strategy is used.

■ 3) Simple Strategies

This is the price process. What does it mean to have a simple strategy? A simple strategy is already quite sophisticated at least for people who are not trained in finance. A simple strategy is a rebalancing strategy in a certain way. For example, you have committed to have 60% of one asset, 40% of the other asset, you fix these proportions over time which means that, when the prices fluctuate,

you have to adjust the units in order to keep your wealth proportions. If everything goes fine and, for example, prices fluctuate around a trend like this, on average you sell when prices are high and you buy when prices are low. So every such strategy has this feature, they all have these rebalancing strategies. The only question is which weights to balance.

■ Mean Variance Analysis Catch 22

There are some rules from which you can choose. One rule is the mean variance portfolio selection, and I do not have to explain to you how this figure is achieved. This is the mean variance rule where we compute the mean variance portfolio, here is the tangential portfolio and this is a certain way of splitting up your investments among those assets.

The other rule is the capital asset pricing rule. The capital asset pricing model is already a huge step from the mean variance, because the capital asset pricing model says suppose everybody does mean variance, what is the consequence of this in equilibrium. The result was given by Sharp, Mossin and Lintner. If everybody does the mean variance rule, then everybody buys the market portfolio which means that the lambdas you have seen before are equal to the market capitalization, divided by total market capitalization.

The second rule which you can choose is the CAPM rule but be aware that there are two different rules. In heterogeneous populations where not everybody plays the mean variance rule, the CAPM rule is something different from the mean variance rule, because Markowitz rule and Sharp, Mostyn and Lindner's rule are connected in the sense if everybody plays the mean variance, then the mean variance is the capital asset price rule.

What is the Catch 22 of mean variance analysis? If you do the mean variance portfolio, by then you have bought a mean variance efficient portfolio, however you are under-diversified. If you do the mean variance portfolio, therefore, you only hold five such assets out of 29 such assets. Now you may say that under-diversification does not mean anything to me; I want to be mean variance efficient. However, be careful as it will eventually turn out to be important. If you buy the market portfolio, you are completely diversified but you are not mean variance efficient, because only if everybody plays the mean variance rule is the market portfolio mean variance efficient. So this is a problem with what you might have learned in your finance classes, which I call the Catch 22 of mean variance analysis.

Something else that you can do, which has already been mentioned, is you can say that variance is not a very good risk measure, so perhaps I should look at value at risk. Value at risk is not a good risk measure, so we have to look at conditional value at risk or expected shortfall. This is taking the conditional expectation to the left of the value at risk point. This is a coherent risk measure in the sense my colleagues in Zurich have defined it. This is then the conditional value at risk portfolio which, based on these data, looks very similar to the mean variance portfolio.

A fourth rule which you can use is you can say I believe in the efficiency of markets, so I do not do any sort of complicated optimization but I simply trust that the prices follow expected dividends, and perhaps I should hold a portfolio which is proportional to these expected dividends. Do the expected cash flow considerations then in some stationary world you will find that expected dividends will be a good approximation for these prices.

This is the expected dividends portfolio and you can also do something which was recommended in the 1970s, which is the maximum growth portfolio. If you want to maximize your growth rate, this is a nice idea, however it is also very under-diversified and very sensitive to the errors in the expectations as we know. This is the growth optimal portfolio.

■ Behavioral Portfolio Rule 1: “Real Man is Unlike Economic Man”

Now we come to behavioral portfolios. This is a very beautiful portfolio which many traders seem to follow. This is called illusionary diversification by Benartzi and Thaler. I always say that this is my grandmother’s portfolio, because when you have very many children, you have to put equal weight on your children. This is putting equal weight on all those assets.

■ Behavioral Portfolio Rule 2: “Real Man is Unlike Economic Man”

You can also do the prospect theory portfolio. This was explained very nicely during the lunch break. The prospect theory has this loss aversion characteristic, the convexity here and the concavity there. It has this weighting of probabilities and so on. We can compute the prospect theory portfolio and here it is, you might think that this is a wise thing to do.

Now you need to think about which portfolio would you choose. There is this dividend process which I have visualized. I did not give you any variance/covariance matrices but I simply visualized it with this chart. There are these portfolio rules and be aware they are in competition with each other. So it depends now on what the others will be responding.

■ My Simple Portfolio Rule

Now I want to put in what is the ecology of this market by asking you. Let us start with mean variance – who is inclined to use mean variance in this competition in this artificial market? [*counts*] It would be nice to have at least one of each! [*laughter*] Who thinks that Freddy Deldaen and my colleagues at the ETH in Zurich have found the trick which is to do the expected shortfall or conditional value at risk portfolio? Who would invest in this portfolio? [*seven*] Next, expected dividends, who thinks Fama got it right? [*26 approx.*] Is it because you have not been there for lunch? [*laughter*] There was so much talk about behavioral things, perhaps the wine distracted you from the presentation. Who thinks the growth optimal strategy is a good idea? [*eight*] Who thinks my grandma got it right to put equal weights? [*five*] Who thinks that with prospect theory Kahneman and Tversky got it right? [*11*] Mark Salmon is one of those. Finally, who thinks that the CAPM strategy is a good one in this type of competitive situation – nobody?

Now we shall do only 100 iterations because my computer is so slow. I will show you what happens if you do many more iterations. Here are the dividends now normalized and, if I push the command button, you see in period zero is the first population of wealth, so I assume that everybody has invested the same amount of wealth according to the strategy. Then we have a wealth selection process. If somebody gains, he has less wealth and then in the next round his strategy will only be managed according to less wealth. What we are looking at is the wealth selection process and what we are following now is the evolution of wealth over time.

Question: You are doing a fixed mixed rebalance to one of these portfolio rules each period?

Thorsten Hens: Yes, it is. We come to the non-simple strategies later. You see that most of you got it right. This Fama expected dividends rule seems to be very important in this.

Question: How did granny do?

Thorsten Hens: Let us see. Equal weights is here, which means equal weights is still better than the growth optimal portfolio and still better than the CAPM. What we learn from this is that in this population, where already many traders use the expected dividends rule, the expected dividends rule is the best rule. Now you might conjecture that, in a different population where everybody uses the prospect theory rule, perhaps the prospect theory rule is the best one. This only tells you that, for this initial population, you get this type of result.

Let us see what happens in more general cases. Let me try to continue my presentation from here. I was surprised by the presentations before. I have given presentations to other leading investment banks and it never happened that they let me present one of my beautiful formulae but, since I have been here this morning and have seen the copula and many variations and other beautiful formulae, I thought that perhaps I should show you the real thing. [laughter] I should show you what is going on in this type of result.

■ The Model

The model we have here is the simple tree model. It is a tree model in a double sense: first, because the uncertainty is driven by a finite number of branches in every period, so a finite number of states in every period, and, secondly, in the sense that we have these dividends being paid off, so they are Lucas trees as they are giving you the apples that you can then eat. What we are looking at is the $\lambda_{i,k,t}$, which is the percentage of wealth which you invest in asset k according to strategy i . The important equation is this one, which is the evolution of wealth. This is the wealth of all traders using strategy i in period t , which, of course, depends on the uncertainty up to period t . How does this determine the wealth in the next period? By some stochastic interest rate in a sense, because what is in the round brackets here are the stochastic interest rates by which your wealth is multiplied from period t to the next period, which is given by your returns. There is nothing new here. This is the dividend and this is the capital gain divided by the price of the previous period. You must have seen this formula many times but it already gives you the most important ingredient of this model, which is the multiplicative wealth process.

■ Equilibrium Prices

The equilibrium is given by this formula and I plug in some of these omegas to say these are stochastic prices, they are not deterministic, but they depend on the λ which in some cases can be simple as before. Depending on t or whatever, they can be adapted to the uncertainty but they also depend on the wealth distribution.

What happens in a typical run is the following situation. Here I have done a typical run where the best strategy again is the expected dividend strategy. However, we have given the best strategy very little income at the beginning, so we have given it only 1 per mil of the total income, so it is only one-tenth of a percent of the income in the market. Initially, some strategies flourish. For example, this is the prospect theory strategy which does very well initially. However, this strategy gradually grows and will eventually, after some period, take over this market. So there is something special with this Fama rule in this type of reasoning. You may say that this is a simple one and does not tell you anything.

The next thing is to look at the mean of those runs, so we have done hundreds of such runs and have computed the mean performance. Again, this is the expected dividends portfolio, this is the prospect theory portfolio. So the simple run was somewhat characteristic for the means. You can also look at the standard deviation, so this is the mean +/- 1 standard deviation, and what was surprising to me, at least, is that there is not much risk in a sense. So the standard deviation bands are very narrow and they do not even widen but at the end they become narrow again. In this type of market, there is a simple normative theory: please choose your portfolio according to the expected relative dividends, which is a fundamental strategy.

Question: Did you ever say how expected dividends were predicted?

Thorsten Hens: No, there is nothing to predict. We assume a stationary process, we even assume an ergodic process, which means that we can use the historic sample mean as the best predictor for the future expectations. This is the very nasty assumption which is able to drive our results.

■ Analysis of the Evolution of Relative Wealth

This is the tool of how to analyze this. You can look at this in terms of relative wealth so, instead of looking at absolute wealth, divide each wealth by total wealth, look at this equation which you have seen before in terms of relatives. This relative wealth is strategy i , this price is divided by the total wealth and the dividends are divided by total wealth. Why do I do this renormalization?

■ Random Dynamical System

Because this gives us the price formula and this gives us a new object which we can analyze mathematically, as this mapping from relative wealth in period t to relative wealth in period $t+1$ is the so-called random dynamical system. It is a dynamical system living on the simplex of dimension of the number of strategies. It is not a deterministic dynamical system but it is a stochastic dynamical system, which we call a random dynamical system. We are fortunate that, during the last 20 or years or so, the mathematics have been discovered to analyse these random dynamical systems. My co-author Klaus Schenk-Hoppe gained a PhD working on these random dynamical systems.

When you go through the analysis of this wealth process in relative terms, you see the key term which is driving the evolution of wealth. The key term is this object here, which is the exponential growth rate of the wealth of strategy i in a population where the population plays strategy n . What is this? This is the expected logarithm where here you have these dividends, here is what strategy i is putting on asset k , and here is what the incumbent strategy, in a sense the incumbent monomorphic strategy n is putting on asset k . So this thing here is the key element by which you can explain the results which you have seen before. Theorem 1 by Evstigneev, me and Klaus Schenk-Hoppe is saying suppose dividends follow in an ergodic process and consider those simple strategies, then there is one such strategy, this λ^* strategy, where you put your wealth according to the relative expected dividends. This strategy is the so-called single survivor, which means, as you have seen in the simulations, that this strategy will eventually hold all market wealth.

Or, to put it into game theoretic terms, imagine that this is a game in a certain sense, imagine there is some other strategy at λ head, then λ^* is going against λ head. Imagine there is λ^* in the market, then λ head is decreasingly against λ^* , or, in a certain sense, λ^* is the dominant strategy of this game.

This, again, is the theorem I once presented to UBS and then somebody told me that this works for Dow Jones data, so does it also work for Swiss data. That is the theorem, right? So this says that whenever this assumption is true, then this theorem holds.

■ Corollaries

What are the corollaries of the theorem? First of all, grandma got it right in a certain sense. You should never use an under-diversified portfolio. Why? Imagine that you omit to put some wealth on a certain asset k and then looking at this growth rate, if you put a zero here, it is trivial to see that there is some other strategy which drives you out of the market, there is some other strategy which goes against you. It only needs to put an epsilon for this asset where you have omitted to put some wealth. Never use an under-diversified strategy, this is fatal for your strategy.

The CAPM strategy, however, is a clever one, it is a passive imitation strategy. If you remember the price formula, according to the CAPM strategy you would like to buy proportionally to the market capitalization. We have assumed there is a supply equal to one, so the market capitalization is the prices here, which means that the capital asset pricing strategy means that, whatever happens in this market, you will follow the winning strategy, because you choose the λ according to the prices. Here is the selection process but you always follow the winning strategy.

If this converges to λ^* , the CAPM strategy, without knowing anything, follows the best winning strategy. So mean variance is very bad from this point of view, because it is under-diversified but CAPM is at least passive; it is following the best strategy as an imitation rule. To put it into other words, if you do a buy and hold strategy of the market portfolio, if you have 10% of the market at the beginning, then you have 10% of the market at the end, because you do not trade. You have a simple buy and hold strategy.

■ Who is right: Classical or Behavioral Finance?

What did our theorem say? Our theorem says that Friedman and Fama got it right, which is that our theorem proved the market selection hypothesis, which Friedman and Fama had put into place long before behavioral finance was developed. They were saying the market selects for rational investors but now we can be a little bit more precise. We have to say in the long run market select for a specific type of rational investors. Note that rationality is not clearly defined if you have a heterogeneous population against which you have to react. However, our theorem says that, in the long run, market select for λ^* , a specific type of rational investors.

Who is right? Behavioral finance says that real men do not behave like economic men but Friedman and Fama would say that real men will be driven out of the market, and this is what we have proved here.

■ Great, but ...

Now we come to the “great, but”. This is not all that easy because the founder of behavioral finance a long time ago, John Maynard Keynes, said that, first of all, in the long run we are all dead, and our result is an asymptotic result for the long run. Even though you are young, you might say that after 100 years if I am evaluated, this may not be a relevant horizon for me. Secondly, markets can remain irrational longer than you can remain solvent, which is also from Keynes. This means that, if you follow this type of strategy and if you wait for the long run like LTCM, who were following some type of fundamental strategies and, they would have been right. However, in the short run, they ran out of money.

The question then is: can the performance be improved by using non-simple strategies, which means by switching strategies. This is now the second stage. We have discovered what is the best simple strategy and perhaps we can do something more clever. Perhaps we can choose the strategies from time to time, so we can switch those strategies and then get some time-dependent, more complex strategies.

■ Note That λ^* Does Not Grow Fastest In All Populations

Coming back to this slide, you can see that it would be a better strategy, first, to follow prospect theory to get the growth out of this market where you have so many prospect theory guys. Mark said over lunch that behavioral finance is popular and everybody does this, so perhaps we should follow prospect theory for a while and only when you discover some other strategy like λ^* has a better advantage, you should switch to some other strategy.

Initially, prospect grows faster than λ^* and this is the so-called “market risk”. The fundamental strategy of Fama has the risk that some other behavioral strategy is growing better if there are many behavioral guys in the market.

■ Credo of Non-Simple Strategies

What is the problem with this point of view? It is a funny problem and nobody has thought through this before, I claim. If you go to this reasoning, what you should do in every period of time is figure out what is the behavioral anomaly of the market. What are the behavioral problems with probabilities, base rule, disposition effect and all these type of things. Then you want to make a clever strategy which is exploiting these behavioral problems of the other traders. This is

the advice behavioral finance gives you and DeBondt and Thaler have been very successful in doing those strategies. They look at market anomalies and they insert a strategy into this market which exploits this anomaly.

■ Catch 22 of Behavioral Strategies

What is the problem with this? It is written in red here. You try to exploit the behavior of others but be sure they try to exploit the behavior of you! This is not a stable situation in a sense because, when you think you are cleverer than someone else because you might live on his behavioral anomalies, he might also think he is cleverer than you and that he can live on your behavioral anomalies.

What is the only solution to this type of Catch 22? The only solution is the Nash equilibrium, because in the Nash equilibrium everybody is playing the best response against everybody else. So only this can be a stable sort of situation. Moreover, we would like to require that the Nash Equilibrium is evolutionary stable, i.e. it cannot be turned over easily by the innovation of new strategies. If you are not playing an evolutionary stable Nash equilibrium, the population is very fragile to certain entrance into the pool of strategies.

■ Nash Equilibrium and “Invasion Risk”: Theorem 2

Our second result says that only λ^* , which you have seen before, is an evolutionary stable Nash equilibrium in this type of game. There is this story, where there is the conclusion that there may be a changing chain of market anomalies. In our theorem, you would phrase it in the following way. Imagine λ^1 is the strategy which is played in the market at one point in time. Then you see here that λ^3 can invade the market, so λ^3 can drive out λ^1 . Now imagine that λ^3 is in the market and you see that λ^2 can drive out λ^3 . Now imagine that λ^2 is in the market, then Farmer & Lo give examples that, again, λ^1 is driving out λ^2 . Therefore, you can have these cycles where you have various strategies. It is like these Escher visual optical illusions where you always go up the stairs but you end up where you were before. Our theorem says that only if λ^* is in the market, this strategy is, first, the Nash equilibrium and, secondly, it is a stable Nash equilibrium in the evolutionary sense. This strategy cannot be driven out by a small fraction of invaders. This is our theorem 2, this fundamental strategy is the unique evolutionary stable strategy.

■ Market Selection Hypothesis: Mutation Risk

This is to remind you what we have seen recently in the data. Here I did something very simple and I should not become famous for this, because Shiller has a similar graph, which however stops here. I just plotted the nominal GDP of the US against the total dividends paid out by the firms in the US, so this somewhat resembles fundamental values. Here is the market capitalization. So the red thing is the prices, the market capitalization. What you see at the beginning of the 1980s is that they were in accordance with each other. Then there was this sudden take-off in the middle of the 1990s and now we are closing the gap – when you travel by Tube they always say “mind the gap”. That is this part here, “mind the gap!”.

How does this confirm our theory? I would say that 1980 to 1990 this was an evolutionary stable situation where, when you only have a small number of invaders in the market, a small fraction of people doing momentum strategies and this type of thing, you will remain in a stable situation. However, evolutionary stability does not tell you what happens if there is a big push, if there is a huge new population coming in which happened here. So there was a big push towards momentum strategies, which was fuelled by new words such as “new economy” and this type of thing. Then we know that this type of situation cannot be stable, because any population mix, which has irrational strategies against those who can do very simple changes, very small population changes, which leads you back by a sequence of population changes, will lead you back to close the gap to the fundamental values.

■ Hints for Portfolio Management

What are the hints for portfolio management? You should be careful with this as it is based on stationarity assumptions and perhaps you should not trust stationarity assumptions too much. If you want to derive a theoretical result, it is hard not to do any stationarity assumptions.

For example, even if you do an evolutionary trading algorithm, in order that this may help you for the future, you implicitly have to assume that there is some sort of stationarity in the data. If the future is totally different from the past, you should not do any type of optimization.

We have assumed the stationarity and ergodic process is already more general than you find in most of the classical finance papers. Based on these assumptions, therefore, we have four recommendations. First, if you are sufficiently patient, for example if you are running a pension fund, an insurance company or whatever, there is an optimal simple strategy, there is no doubt about that and this is λ^* . Secondly, do not use an under-diversified strategy, because you can easily be driven out of the market as we have seen. Thirdly, in the short run the optimal portfolio depends on the ecology of the market, this is unstable because there are all these new strategies which can invade the market. However, a rational market, a λ^* market is stable.

That is my presentation. Now I hand you over to Bart again.

7. Concluding Remarks – Bart Dowling

Thank you one and all for coming. When I first suggested that we should do this conference, I was looked at with a degree of bemusement and shock on behalf of some of the people with whom I deal, as they did not think there were that many of you – people who would tolerate, as Thorsten said, a high degree of mathematics being put to them on such a concentrated basis in one day. However, the good news is that you are here and for that I thank you.

As I said at the outset, I have consciously decided to evolve my role into providing a high-end ‘niche’ or ‘boutique’ facility within the finance industry – what continues to surprise is exactly how large this ‘niche’ is. I hope to make this an annual event and, as in any system dynamic equation, feedback is important. Your ‘feedback’ in the form of attendance is well appreciated and no doubt I anticipate we shall have an even larger gathering next year.

In Germany, this report should be read as though Merrill Lynch has acted as a member of a consortium which has underwritten the most recent offering of securities during the last five years for companies covered in this report and holds 1% or more of the share capital of such companies.

The analyst(s) responsible for covering the securities in this report receive compensation based upon, among other factors, the overall profitability of Merrill Lynch, including profits derived from investment banking revenues.

OPINION KEY: Opinions include a Volatility Risk Rating, Intermediate-Term and Long-Term Investment Ratings and an Income Rating. **VOLATILITY RISK RATINGS**, indicators of potential price fluctuation, are: A – Low, B – Average, C – Above Average, D – High. **INTERMEDIATE-TERM INVESTMENT RATINGS**, indicators of expected total return (price appreciation plus yield) within the 12-month period from the date of the initial rating, are: 1 – Strong Buy (minimum 20% – more for High Risk securities); 2 – Buy (minimum 10%); 3 – Neutral (0- 10%); 4 – Reduce/Sell (negative return); 6 – No Rating. **LONG-TERM INVESTMENT RATINGS**, indicators of fundamental company factors demonstrating potential total return for the 3-year period from the date of the initial rating, are: 1 – Strong Buy (aggregate minimum 40%); 2 – Buy (aggregate minimum 20%); 3 – Neutral (aggregate 0-20%); 4 – Reduce/Sell (negative return); 6 – No Rating. **INCOME RATINGS**, indicators of potential cash dividends, are: 7 – same/higher (dividend considered to be secure); 8 – same/lower (dividend not considered be secure); and 9 – pays no cash dividend.

Copyright 2002 Merrill Lynch, Pierce, Fenner & Smith Incorporated (MLPF&S). All rights reserved. Any unauthorized use or disclosure is prohibited. This report has been prepared and issued by MLPF&S and/or one of its affiliates and has been approved for publication in the United Kingdom by Merrill Lynch, Pierce, Fenner & Smith Limited, which is regulated by the FSA; has been considered and distributed in Australia by Merrill Lynch Equities (Australia) Limited (ACN 006 276 795), a licensed securities dealer under the Australian Corporations Law; is distributed in Hong Kong by Merrill Lynch (Asia Pacific) Ltd, which is regulated by the Hong Kong SFC; and is distributed in Singapore by Merrill Lynch International Bank Ltd (Merchant Bank) and Merrill Lynch (Singapore) Pte Ltd, which are regulated by the Monetary Authority of Singapore. The information herein was obtained from various sources; we do not guarantee its accuracy or completeness. Additional information available.

This research report is prepared for general circulation and is circulated for general information only. It does not have regard to the specific investment objectives, financial situation and the particular needs of any specific person who may receive this report. Investors should seek financial advice regarding the appropriateness of investing in any securities or investment strategies discussed or recommended in this report and should understand that statements regarding future prospects may not be realized. Investors should note that income from such securities, if any, may fluctuate and that each security's price or value may rise or fall. Accordingly, investors may receive back less than originally invested. Past performance is not necessarily a guide to future performance.

Neither the information nor any opinion expressed constitutes an offer to buy or sell any securities or options or futures contracts.

Foreign currency rates of exchange may adversely affect the value, price or income of any security or related investment mentioned in this report. In addition, investors in securities such as ADRs, whose values are influenced by the currency of the underlying security, effectively assume currency risk.

